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Multidimensional private information, market structure, and insurance markets

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We investigate whether selection based on multidimensional private information in risks and risk preferences can, under different market structures, result in a negative correlation between insurance coverage and ex post *realization of risk. We show that, under perfect competition, selection based on multidimensional private information does not result in the negative correlation property, unless there is a sufficiently high loading factor. However, it is possible to generate the negative correlation property under monopoly when risk and risk preference types are sufficiently negative dependent. We also clarify the connections between important concepts such as adverse/advantageous selection and positive/negative correlation property.*

1. Introduction

■ The classic asymmetric information models of insurance pioneered by Arrow (1963), Pauly (1974), Rothschild and Stiglitz (1976), and Wilson (1977) assume that potential insurance buyers have *one*-dimensional private information regarding their risk type. These models predict a positive correlation between insurance coverage and *ex post* realizations of losses. The reason is *ex ante* adverse selection, namely, that the "bad risks" (i.e., those relatively likely to suffer a loss) have a higher willingness to pay for insurance; and allowing for *ex post* moral hazard only strengthens the positive correlation between coverage and *ex post* losses. This "positive correlation property" of the classic asymmetric information models forms the basis for empirical tests of asymmetric information in several recent articles (see Chiappori and Salanié, 2000).

However, the results from a growing empirical literature testing for the correlation between insurance coverage and *ex post* realization of risks are mixed and vary by market. In an auto insurance market, Chiappori and Salanié (2000) find that the accident rate for young French

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drivers who choose comprehensive automobile insurance is not statistically different from those opting for the legal minimum coverage, after controlling for consumers' characteristics observable to the automobile insurers. In contrast, Cohen (2005), using data from an online Israeli insurer, finds that new auto insurance customers choosing a low deductible contract tend to have more accidents, leading to higher total losses for the insurer.[1] In the life insurance market, Cawley and Philipson (1999) find that the mortality rate of US males who purchase life insurance is below that of the uninsured, even when controlling for many factors such as income that may be correlated with life expectancy.[2] For the long term care (LTC) insurance market, Finkelstein and McGarry (2006), using panel data from a sample of Americans born before 1923 (the AHEAD study), find no statistically significant correlation between their LTC coverage in 1995 and their use of nursing home care between 1995–2000, even after controlling for the insurers' assessment of a person's risk type. Moreover, when Finkelstein and McGarry (2006) use whether respondents undertake various types of preventive healthcare as a proxy for risk aversion, they find that people who are more risk averse by this measure are both more likely to own LTC insurance and less likely to enter a nursing home. In an annuity insurance market, Finkelstein and Poterba (2004) find systematic relationships between the *ex post* mortality and the annuity characteristics, such as the timing of payments and the possibility of payments to the annuitants' estate, but they do not find evidence of substantive mortality differences by annuity size. For the Medigap insurance market, Fang, Keane, and Silverman (2008) find that, conditional on controls for Medigap prices, those with Medigap spend on average $4000 less on medical care than those without, providing a strong evidence for the negative correlation between Medigap purchase and *ex post* realization of risk.

These empirical findings fueled an emerging literature on the possibility that *multidimensional* private information may lead to what has been called "advantageous selection." [3] The formal theoretical literature is sparse. de Meza and Webb (2001) postulate a model in which individuals differ in their risk preferences, which they refer to as "timid" and "bold" types. They assume that more timid types may lower their risk exposure through increased insurance purchase and greater precautionary effort to reduce risks. They show that, in the presence of administrative costs in processing claims and issuing policies, there exists a pure-strategy, partial pooling, subgame-perfect Nash equilibrium in the insurance market that exhibits the negative correlation property. Thus, failure to condition on risk aversion may then mask the positive correlation between insurance coverage and *ex post* risk predicted by one-dimensional models. Following de Meza and Webb (2001), the existing literature points to risk preferences as the primary suspect behind advantageous selection. In general, however, *any* private information could function as a source of advantageous selection if it is positively correlated with insurance coverage and at the same time negatively correlated with risk. Finkelstein and McGarry (2006) argue that their findings on the LTC insurance market is consistent with multidimensional private information and advantageous selection based on risk aversion. In fact, their findings suggest that, on net, adverse selection based on risk and advantageous selection based on risk aversion roughly cancel out in the LTC insurance market. Fang, Keane, and Silverman (2008) find that, for Medigap insurance market, risk preferences do not appear as a source of advantageous selection, but cognitive ability is particularly important.

However, to the best of our knowledge, the precise conditions under which whether selection based on multidimensional private information may generate *in equilibrium* a positive or negative

[1] Others have examined the evidence of asymmetric information in the choice of insurance contracts such as deductibles and copayments, etc. For example, Puelz and Snow (1994) study automobile collision insurance and argue that, in an adverse selection equilibrium, individuals with lower risk will choose a contract with a higher deductible, and contracts with higher deductibles should be associated with lower average prices for coverage. They find evidence in support of each of these predictions using data from an automobile insurer in Georgia. However, see Chiappori and Salanié (2000) and Dionne, Gouriéroux, and Vanasse (2001) for critiques of the Puelz and Snow study.

[2] See He (2009) for a reexamination of the evidence.

[3] The first description of this phenomenon in the economics literature appears to be Hemenway (1990), who used the term "propitious selection."

correlation between insurance purchase and *ex post* realization of risk is still unknown. Most of the existing articles that invoked the possibility of multidimensional private information as a possible explanation for the empirical findings discussed above rely on partial equilibrium intuition (much in the spirit of Hemenway, 1990). An important exception is Chiappori, Jullien, Salanié, and Salanié (2006, henceforth, CJSS), which argues that in a competitive insurance market, the positive correlation property is a general implication of insurance models with asymmetric information, even when the private information is multidimensional in risks and risk preferences. The key assumptions are consumer rationality and a condition which they refer to as "nonincreasing profit" (NIP) condition—that is, the per-contract expected profit does not increase with the coverage of the contract.[4] CJSS's approach is general and elegant, and they prove their results using revealed preference and the NIP condition. However, the nonincreasing profit condition is not a primitive condition; thus, whether it holds in equilibrium in environments where the market may not be competitive and where loading costs for offering insurance exist is still an open question.

The goal of this article is to help fill in this gap. We present a simple model of insurance market where consumers have multidimensional private information in risk and risk preference types, and investigate whether selection based on multidimensional private information can, under different market structures, result in negative correlation in equilibrium between insurance coverage and *ex post* realization of risk. We show that if the insurance market is perfectly competitive, selection based on multidimensional private information does *not* generate the negative correlation property in equilibrium unless there is a sufficiently high loading factor, possibly because of administrative or marketing costs. If the insurance market is monopolistic, however, we show that it is possible to generate the negative correlation property in equilibrium when consumers' risk type and risk preference type are sufficiently negative dependent, a notion we formalize using the concept of copula. It should be noted that the fully specified model considered in our article is not as general as that in CJSS; our contribution is to state the connections between the primitives of the model (including multidmensional private information and market structure) and the positive or negative correlation property in a transparent way.

The remainder of the article is structured as follows. In Section 2, we provide a detailed discussion of the related literature. In Section 3, we describe our model environment in which consumers are heterogeneous in both risk and risk preference types. In Section 4, we consider the perfectly competitive market structure. In Section 5, we analyze the monopolistic market.[5] In Section 6, we clarify the confusions in this growing literature about the connections between some of the important concepts such as adverse/advantageous selection and positive/negative correlation property. In Section 7, we partially endogenize the contract space and again show that our results for the single contract case derived in Sections 4 and 5 continue to hold with natural and mild generalizations of the assumptions imposed in Section 3. In Section 8, we summarize our main findings and suggest directions for future research. All proofs are relegated to an Appendix.

2. Related literature

■ To the extent that our article investigates on whether the positive correlation property is robust to environments with multidimensional consumer heterogeneity, it is most related to CJSS (2006) and de Meza and Webb (2017). CJSS argue that, as long as consumers are rational and the per-contract expected profit does not increase with the coverage of the contract (which they refer to as "nonincreasing profit" [NIP] condition), then the positive correlation property is robust to multidimensional private information. This conclusion is similar to our results for the competitive insurance market presented in Propositions 1–2 and Proposition 7. Their results are proved using the revealed preference argument implied by the hypothesized consumer rationality

[4] We will discuss the connection between our results and theirs in Section 4.

[5] In an online web Appendix, we show that our results for the monopolistic market structure can be generalized to a version of an imperfectly competitive market structure.

and the nonincreasing profit condition assumed on the supply side, and as such, they do not have to exploit the full set of equilibrium restrictions. In contrast, we exploit the full set of the equilibrium restrictions and as a result, our positive correlation predictions for the competitive insurance market are sharper for the case of proportional loading factor. We will provide more details of the comparison when we discuss Proposition 2. Also related, de Meza and Webb (2017) provide an insightful discussion that the positive correlation test is not a valid test to distinguish asymmetric information from symmetric information environment. The reason is that under symmetric information, the only insurance purchased by consumers will be the one with full coverage, unless the claim processing costs (or other loading factors) are formally modelled. The results in our article are more relevant in distinguishing multidimensional versus one-dimensional private information models of insurance as opposed to symmetric versus asymmetric information models of insurance.

Our article is also related to a recent literature that attempts to analyze the selection markets with potentially multidimensional private information. Einav, Finkelstein, and Cullen (2010) propose an approach to conduct empirical welfare analysis in insurance markets based on directly estimating the demand and average (and marginal) cost curves using exogenous variations in prices.[6] Based on their graphical analysis of the demand and cost curves for the selection market, where the defining feature is that insurers' costs depend on which consumers purchase their products and hence are endogenous to price, they also argue that the slope of the estimated marginal cost curve provides a direct test of the existence and the nature of selection, that is, whether the selection is adverse or advantageous. Specifically, they argue that a rejection of the null hypothesis of a constant marginal cost curve is a rejection of the null hypothesis of no selection, whereas the selection is adverse (respectively, advantageous) if the marginal cost is increasing (respectively, decreasing) in price. They also emphasize that an attractive feature of their approach of relying only on the estimated demand and cost curves is that "it does not require the researcher to make (often difficult-to-test) assumptions about consumers' preferences or the nature of *ex ante* information" (Einav, Finkelstein, and Cullen, 2010). In Section 6, we show that an important limitation of their approach is that typically, the marginal cost curve is *nonmonotonic* when consumer heterogeneity is multidimensional.[7] In fact, in our setting with two-dimensional private information in risk and risk aversion, it is monotonic only when the two dimensions are perfectly correlated. Therefore, depending on the range of the price variations available in the data that is used to estimate the demand and cost curves, it is likely the estimated cost curve only reflects the nature of the selection–adverse or advantageous–locally.[8] We also provide an example (Example 4) in which the nature of the local selection being advantageous at the equilibrium price level does not imply a negative correlation between insurance purchase and *ex post* realization of risk in equilibrium.

We explicitly model consumers' multidimensional heterogeneity in this article. The intuition for why the marginal cost curve is unlikely to be monotonically increasing in the market size in our model can be easily explained for the case of bounded consumer type space. Consider an environment where consumers' willingness to pay for insurance is increasing in their risk type m and risk-aversion type λ. Suppose that risk type and risk-aversion type are bounded in $[\underline{m}, \overline{m}]$ and $[\underline{\lambda}, \overline{\lambda}]$, respectively. Then, the marginal cost of insurance when the market size is close to 0 will be close to $\overline{m}$, and the marginal cost when the market size is close to 1 will be close to $\underline{m}$. That is, the marginal cost curve must always have at least one decreasing segment! Another

[6] See also Einav, Finkelstein, and Levin (2010), Einav and Finkelstein (2011), and Chetty and Finkelstein (2013) for related discussions of the demand and cost analysis of selection markets.

[7] Einav and Finkelstein (2011) are aware of the nonmonotonicity issue of the marginal cost curve, as they stated : "More generally, once we allow for preference heterogeneity, the marginal cost curve needs not be monotone. However, for simplicity and clarity we focus our discussion on the polar cases of monotone cost curves." (footnote 7).

[8] In empirical applications, the range of the exogenous price variations is often quite limited. For example, Einav, Finkelstein, and Cullen (2010) have a total of six price levels, (or three price levels if one only considers those with somewhat large numbers of consumers).

benefit of modelling consumers' multidimensional heterogeneity explicitly is that it allows us to examine how the market outcome changes when the dependence structure of individuals' multidimensional heterogeneity varies. We use the concept of copula to parameterize the degree of dependence between the consumers' multidimensional types.

Mahoney and Weyl (2017) build and analyze a model of imperfect competition in selection markets. They parameterize the degree of both market power and selection, and use graphical price-theoretic reasoning to analyze the interactions between selection and imperfect competition. Their parameterization of selection follows Einav, Finkelstein, and Cullen (2010) by hypothesizing whether the marginal cost curve is either upward or downward sloping.

Azevedo and Gottlieb (2017) propose an equilibrium concept for competitive insurance market where consumers may have multidimensional heterogeneity, and insurance companies compete for consumers by choosing contracts from a compact space and setting their corresponding prices. Their equilibrium concept, which relies on perturbations, guarantees existence. Veiga and Weyl (2016) instead study the incentives for a monopolistic insurer in its choice of insurance quality facing consumers with multidimensional heterogeneity. They derive a condition of the optimal insurance quality to emphasize that the sorting incentives of the monopolist is the ratio of two terms: the numerator is the covariance among marginal consumers between the marginal willingness to pay for quality and the cost for the firm, and the denominator is marginal consumer surplus, which measures market power. The analysis of Veiga and Weyl (2016) focuses on the marginal consumers, and does not analyze correlation between insurance purchase and *ex post* realization of risk, which is about the average insurance buyers and nonbuyers; and they also focus on the monopolistic market structure, whereas our article highlights the interactions between multidimensional heterogeneity and market structure.[9] In the basic model of our article, the quality of insurance is assumed to be exogenously fixed, and we focus on the determination of premium; whereas Veiga and Weyl (2016) focus on how the monopolistic insurer determines the profit-maximizing quality of insurance. Neither Azevedo and Gottlieb (2017) nor Veiga and Weyl (2016) analyze whether multidimensional consumer heterogeneity can generate a negative correlation between insurance purchase and *ex post* realization of risk in equilibrium.[10]

3. The model

Consumers. There is a continuum of consumers with heterogeneous types indexed by $\theta \equiv (m, \lambda)$, where $m \in [\underline{m}, \overline{m}]$ with $0 \leq \underline{m} < \overline{m} < \infty$ denotes consumer's risk type, and $\lambda \in [\underline{\lambda}, \overline{\lambda}]$ with $0 < \underline{\lambda} < \overline{\lambda} < \infty$ can be interpreted as any other characteristics of the consumer that may be related to his/her risk preference.[11] As a notational convention, we use M and Λ, respectively, to denote the random variables for risk type and risk preference type, and their lowercase counterparts as their realizations. In the population, consumers' type, (m, λ), is assumed to be drawn from joint CDF $H(m, \lambda)$, and we denote the marginal CDF of M and Λ by $F(m)$ and $G(\lambda)$, respectively. We use $h(m, \lambda)$, $f(m)$, and $g(\lambda)$ to denote the corresponding joint and marginal density functions of (M, Λ), M, and Λ, respectively. We assume that the marginal distribution of M is such that M has finite mean, denoted by $\mathbb{E}[M]$. We allow *dependence* between M and Λ in this article, and we will discuss the form of the dependence in detail in Section 5.

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[9] Also related to our article, Weyl and Veiga (2014) offer a quantitative strengthening of the notion of affiliation for multidimensional random vectors that is useful to relate dependence between risk types and risk preferences to the direction of selection.

[10] It should be noted that Azevedo and Gottlieb (2017) introduce a notion of "intensive margin selection coefficient" that measures the difference between the marginal changes of the premium and the cost of insuring the marginal consumers, both with respect to the insurance coverage. They suggest that this notion is related to the positive correlation test. We will discuss its connection with our results in Sections 6 and 7 below.

[11] We assume that m and λ are bounded above by $\overline{m}$ and $\overline{\lambda}$, respectively, for the simplicity in describing some of the intuitions for our results. Most of our results remain qualitatively unchanged if risk type and risk preference type are supported on semiinfinite intervals, that is, $(m, \lambda) \in [\underline{m}, \infty) \times [\underline{\lambda}, \infty)$. In particular, see our discussion on the differences caused by the semiinfinite support in footnote 32 .

Insurance contract. Consumers decide whether or not to purchase insurance. In the basic model, we assume that insurance firms are regulated in the sense that they can *only* provide insurance with quality $x \in (0, 1]$, where a higher x indicates a contract with better coverage. Note that we assume that the insurance coverage quality x is *not* a choice variable for the firms. As such, our setup is in the spirit of Akerlof (1970), where insurance contract is exogenously given, rather than Rothschild and Stiglitz (1976) where the insurance quality x is endogenously chosen. Although exogenous contract space is an important restriction, it is useful to point out that this approach is adopted in most of the recent applied literature. For instance, Einav, Finkelstein, and Cullen (2010) assume that a consumer either buys a homogeneous insurance policy or does not buy any coverage. Similarly, Handel, Hendel, and Whinston (2015) assume that there are only two options: a low-coverage contract and a high-coverage contract.

The cost to the insurance firms, not including loading costs (e.g., contract processing cost and other administrative costs), for providing quality-x insurance to a type-$\theta \equiv (m, \lambda)$ consumer, denoted by $C(\theta; x)$, is increasing in the consumer's risk type m and the coverage quality x. Note that it does *not* depend on the consumer's risk preference type λ. In particular, we let $C(\theta; x) = x \cdot m$.

Consumer preference. Type-$\theta \equiv (m, \lambda)$ consumer derives utility $U(\theta; x, p)$ from purchasing a contract with quality $x \in [0, 1]$ and premium $p \in [0, \infty)$. Without loss of generality, we assume that consumers derive zero utility from the null contract $(x, p) = (0, 0)$, that is, $U(\theta; 0, 0) \equiv 0$ for all θ.

Assumption 1. $\partial U/\partial p < 0$. Moreover, fixing $x \in (0, 1]$, there exist p' and p'' such that $U(\theta; x, p') < 0 < U(\theta; x, p'')$.

Denote type-$\theta \equiv (m, \lambda)$ consumer's willingness to pay (WTP) for an insurance policy with quality x by $v(\theta; x)$. By definition, $v(\theta; x)$ is the solution to

$$U(\theta; x, v) = U(\theta; 0, 0) = 0. \tag{1}$$

Assumption 1 guarantees the existence and uniqueness of such a solution for all $x \in (0, 1]$. We make the following assumptions on $v(\cdot)$:

Assumption 2. $\partial v/\partial x > 0$, $\partial v/\partial m > 0$, and $\partial v/\partial \lambda > 0$.

Assumption 2 simply says that consumer's WTP for insurance is increasing in her risk type, in her risk preference type, and the quality of the contract coverage.

Assumption 3. $v(\theta; x) > C(\theta; x) \equiv x \cdot m$, for all θ and any $x \in (0, 1]$.

Assumption 3 holds for many economic framework of insurance as long as individuals are risk-averse.[12] The difference between the WTP for insurance $v(\theta; x)$ and $C(\theta; x)$ is commonly referred to as the *risk premium* for type-(m, λ) consumer.

Facing a premium p for insurance coverage x, a type-(m, λ) consumer purchases insurance if and only if $U(\theta; x, p) \geq U(\theta; 0, 0) \equiv 0$, or equivalently, $p \leq v(\theta; x)$. We use

$$\mathcal{B}(p) \equiv \{\theta : v(\theta; x) \geq p\} \tag{2}$$

to denote the set of consumers whose WTP for the insurance exceeds the premium, and thus they are the set of *buyers*; and use

$$\mathcal{N}B(p) \equiv \{\theta : v(\theta; x) < p\} \tag{3}$$

[12] In an environment with heterogeneity in *ex post* moral hazard, such as that studied in Einav et al. (2013), it is possible that $v(\theta; x)$ does not always exceed $C(\theta; x)$, as shown in Example 3 of Azevedo and Gottlieb (2017). Moral hazard may also indirectly lead to a violation of Assumption 2 if consumers face budget constraints.

to denote the set of *nonbuyers* at price p. Assumption 2 implies that the iso-WTP curve is downward sloping in the (m, λ) space for $x \in (0, 1]$ and the set of consumers above (respectively, below) the curve is the set of buyers (respectively, nonbuyers). In addition, Assumption 1 together with $U(\theta; 0, 0) = 0$ implies that consumers' WTP for zero coverage is zero, that is, $v(\theta; 0) = 0$ for all θ.

Remark 1. In practice, firms can charge premiums based on observable characteristics. In this article, we simplify our analysis by assuming the observed characteristics are the same across all consumers. It is useful to think of our analysis as being within the consumers of a particular risk classification class. This simplification allows us to focus on the comparison between multidimensional and one-dimensional private information.

Remark 2. We would like to emphasize that the key distinction between one-dimensional and multidimensional private information models is whether the ranking of consumers by their WTP is perfectly aligned with the ranking by their costs. Assumption 2 implies that the two rankings are *not necessarily* perfectly aligned in our model, whereas they are always perfectly aligned in the classic asymmetric information models of insurance with one-dimensional private information in risk types. It is in this sense that our model is "multidimensional." It should also be noted that, at a technical level, it is always possible to encode multidimensional types in a single-dimensional variable, so, as always, the content of the multidimensional signals depends on additional assumptions made about the type space.[13]

Example 1 (Binary States). Each consumer has initial wealth y and is subject to a possible loss $\omega \in (0, y)$ with probability κ. The consumer can purchase an insurance contract to cover a fraction x of the loss if it occurs. Let $u(\cdot; \lambda)$ be consumer's Bernoulli utility function, where λ is the risk preference parameter. Let $m \equiv \kappa\omega$. Then, the expected cost to the insurance firm for insuring type-(m, λ) consumer is $C(m, \lambda; x) = x\kappa\omega = xm$. Consumers' net expected utility from purchasing an insurance (x, p) is

$$U(\theta; x, p) \equiv \left[\frac{m}{\omega}u(y - p - (1 - x)\omega; \lambda) + \left(1 - \frac{m}{\omega}\right)u(y - p; \lambda)\right] - \left[\frac{m}{\omega}u(y - \omega; \lambda) + \left(1 - \frac{m}{\omega}\right)u(y; \lambda)\right].$$

It is straightforward to verify that Assumption 1 is satisfied and $U(\theta, 0, 0) = 0$ for all θ. Consumers' WTP for insurance of coverage x is determined by $U(\theta; x, v) = 0$, or equivalently,

$$\frac{m}{\omega}u(y - v - (1 - x)\omega; \lambda) + \left(1 - \frac{m}{\omega}\right)u(y - v; \lambda) = \frac{m}{\omega}u(y - \omega; \lambda) + \left(1 - \frac{m}{\omega}\right)u(y; \lambda).$$

By the Implicit Function Theorem, it can be verified that $\partial v/\partial x > 0$ and $\partial v/\partial m > 0$. In addition, $\partial v/\partial \lambda > 0$ holds if λ orders types according to their risk aversion in the sense of Pratt (1964):

$$\lambda_1 > \lambda_0 \Rightarrow \frac{u''(x, \lambda_1)}{u'(x, \lambda_1)} \geq \frac{u''(x, \lambda_0)}{u'(x, \lambda_0)} \; \forall x.$$

Therefore, Assumption 2 is satisfied. Last, the concavity of $u(\cdot; \lambda)$ implies that $v(\theta; x) > xm = C(\theta; x)$, and hence Assumption 3 is also satisfied.

[13] Multidimensional types can always be encoded in a single-dimensional variable using the inverse Peano function (e.g., Sagan, 1984) and other methods. The difficulty of such a one-dimensional representation of an intrinsically multidimensional problem, however, is that we could not impose reasonable restrictions on the information structure, such as types being drawn from a continuous distribution. Similar issues concerning the representation of multidimensional information with single-dimensional messages have been discussed in the mechanism design literature (see, e.g., Mount and Reiter, 1974). Also see Fang and Morris (2006) for similar discussions.

Example 2 (CARA and Normal Shocks). Consumers have initial wealth y and may experience a medical expenditure $Z \sim \mathcal{N}(m, \sigma^2)$. A consumer has constant absolute risk aversion (CARA) Bernoulli utility $u(y) = -\exp(-\lambda y)$, where λ is consumer's constant absolute risk aversion. With CARA Bernoulli utility, it is without loss of generality to measure consumer's utility by his certainty equivalent, that is,

$$U(\theta; x, p) \equiv xm + \frac{x(2-x)}{2}\sigma^2\lambda - p.$$

Solving equation (1) for v yields,

$$v(\theta; x) = xm + \frac{x(2-x)}{2}\sigma^2\lambda = C(\theta; x) + \frac{x(2-x)}{2}\sigma^2\lambda. \tag{4}$$

It can be verified that Assumptions 1–3 are satisfied in Examples 1 and 2.[14]

4. Competitive insurance market

■ In a competitive insurance market, insurance firms choose a premium p for insurance coverage with the given quality x to compete for consumers. A firm's profit at premium p from offering insurance with coverage x, if there is no loading cost of offering insurance, is given by[15]:

$$\pi(p) = \int_{\theta \in \mathcal{B}(p)} (p - xm) dH(m, \lambda). \tag{5}$$

Denote p^* as the equilibrium price under perfect competition which, in the absence of loading costs, is simply determined by:

$$\pi(p^*) = 0. \tag{6}$$

Remark 3. Equation (6) may have multiple solutions. Any premium greater than $v(\overline{m}, \overline{\lambda}; x)$ satisfies (6) because $\mathcal{B}(p)$ is empty for any $p > v(\overline{m}, \overline{\lambda}; x)$. If consumers' risk premium is sufficiently large, it is also possible that the equilibrium premium is less than $v(\underline{m}, \underline{\lambda}; x)$; in such a scenario, all consumers purchase insurance. Because our goal is to compare the average risk of the consumers with insurance and those without, we assume in the rest of the article that there exists at least one equilibrium with premium p^* that lies strictly between $v(\underline{m}, \underline{\lambda}; x)$ and $v(\overline{m}, \overline{\lambda}; x)$ so that the sets $\mathcal{B}(p^*)$ and $\mathcal{NB}(p^*)$are of positive measures to ensure the conditional expectations (7) and (8) below are both well defined.

Fixing the market price of the insurance $p \in (v(\underline{m}, \underline{\lambda}; x), v(\overline{m}, \overline{\lambda}; x))$, the average *ex post* realization of risk among those who purchase insurance is:

$$\mathbb{E}[M|\mathcal{B}(p)] = \frac{\int_{\theta \in \mathcal{B}(p)} m dH(m, \lambda)}{\int_{\theta \in \mathcal{B}(p)} dH(m, \lambda)}, \tag{7}$$

where the denominator is the measure of the insurance coverage penetration, and the numerator is the total cost realization of the insured. Similarly, the average *ex post* realization of risk among those who do not purchase insurance is

$$\mathbb{E}[M|\mathcal{N}B(p)] = \frac{\int_{\theta \in \mathcal{N}B(p)} m dH(m, \lambda)}{\int_{\theta \in \mathcal{NB}(p)} dH(m, \lambda)}. \tag{8}$$

[14] See the online web Appendix A for the details of the proof that Examples 1 and 2 satisfy Assumptions 1–3.

[15] We will consider how loading costs affect our results below.

It is also useful to define the average *ex post* realization of the risk for the entire population:

$$\mathbb{E}[M] = \int_{\underline{\lambda}}^{\overline{\lambda}} \int_{\underline{m}}^{\overline{m}} m dH(m, \lambda)$$
$$= \int_{\theta \in \mathcal{B}(p)} dH(m, \lambda) \mathbb{E}[m | \mathcal{B}(p)] + \int_{\theta \in \mathcal{N}B(p)} dH(m, \lambda) \mathbb{E}[m | \mathcal{N}B(p)]. \quad (9)$$

Definition 1 (Positive and Negative Correlation Property). The insurance market exhibits *positive* correlation propertyin equilibrium if $\mathbb{E}[M|\mathcal{B}(p^*)] > \mathbb{E}[M|\mathcal{N}B(p^*)]$, and it exhibits *negative* correlation propertyif $\mathbb{E}[M|\mathcal{B}(p^*)] < \mathbb{E}[M|\mathcal{N}B(p^*)]$, where the two terms are defined in (7) and (8), respectively.

Note that Definition 1 defines the positive and negative correlation property for the case of positive coverage versus zero coverage. It is straightforward to generalize our results below to the case where the comparisons are between the expected cost realizations of high versus low coverage. In Section 7, we will formally generalize the definition of positive and negative correlation property when there are contracts with multiple levels of coverages.

Proposition 1 (Positive Correlation Property Always Holds in Competitive Equilibrium without Loadings). Suppose Assumptions 1, 2, and 3 are satisfied and that the equilibrium price p^* is such that the measure of buyers and nonbuyers are both strictly positive. Then, positive correlation property always holds in equilibrium if the insurance market is perfectly competitive and there are no loadings.

Note that Proposition 1 states that negative correlation property will not emerge in a competitive insurance market without loadings, *regardless* of the dependence structure between risk type and risk preference type. The intuition for the result is in fact very simple. If there were a negative correlation between insurance purchase and *ex post* risk realizations, then in a competitive insurance market, the equilibrium premium must be equal to the expected risk realization of the insured, which is lower than that of the uninsured under negative correlation property. However, if the equilibrium premium were indeed lower than the expected risk realization of the uninsured, it must also be lower than their average WTP under Assumption 3. This in turn implies that at the equilibrium premium, some of the uninsured must prefer to purchase insurance as well, which is a contradiction. The following simple three-type example illustrates the aforementioned intuition.

Example 3 (Illustrative Three-Type Example). Suppose that there are three types of consumers in the population. For simplicity, we will describe their types by the combinations of their cost of coverage and WTP for insurance: (c_1, v_1), (c_2, v_2), and (c_3, v_3). Assumption 3 implies that $v_j > c_j$ for $j \in \{1, 2, 3\}$. Let q_j denote the probability that a consumer is of type (c_j, v_j), $j \in \{1, 2, 3\}$, in the population. Suppose that $c_1 < c_2 < c_3$.

In the standard one-dimensional private information model, consumers differ only regarding their risk types. Hence, the discrete analog of Assumption 2 would imply that $v_1 < v_2 < v_3$. Thus, it is immediate that the positive correlation property must hold in any competitive equilibrium.

In a multidimensional private information model, the order of consumers' WTP may differ from the order of their risk types (or their costs of coverages) due to the possibility that consumers with lower risk may be more risk averse. In order for a pure-strategy competitive equilibrium to exhibit the negative correlation property, there are only three possibilities[16]: (i) only type (c_1, v_1) consumers buy coverage in equilibrium at a premium $p^* = c_1$; (ii) type (c_1, v_1) and type

[16] It is straightforward to generalize the argument to allow for mixed strategies.

(c_2, v_2) consumers purchase coverage in equilibrium at a premium $p^* = \frac{q_1}{q_1+q_2}c_1 + \frac{q_2}{q_1+q_2}c_2$, and type (c_3, v_3) consumers remain uninsured; (iii) type (c_1, v_1) and type (c_3, v_3) consumers purchase coverage in equilibrium at a premium $p^* = \frac{q_1}{q_1+q_3}c_1 + \frac{q_1}{q_1+q_3}c_3$, and type (c_2, v_2) consumers remain uninsured. We show that none of the cases are possible.

For case (i), equilibrium requires type (c_1, v_1) consumers to prefer to buy coverage and type (c_3, v_3) consumers to prefer to remain uninsured, that is,

$$v_1 \geq p^* = c_1 \geq v_3.$$

The above inequality, together with $v_3 > c_3$, implies immediately that $c_1 > c_3$, a contradiction.

Similarly, for case (ii) to constitute an equilibrium, we must have that type (c_2, v_2) consumers purchase the policy and type (c_3, v_3) consumers choose to opt out, that is,

$$v_2 \geq p^* = \frac{q_1}{q_1+q_2}c_1 + \frac{q_2}{q_1+q_2}c_2 \geq v_3.$$

The above inequality, together with $v_3 > c_3$, implies immediately that $\frac{q_1}{q_1+q_2}c_1 + \frac{q_2}{q_1+q_2}c_2 > c_3$, which cannot be satisfied due to the postulated $c_1 < c_2 < c_3$.

For case (iii), in order for the equilibrium to have negative correlation property, we need to have

$$p^* = \frac{q_1}{q_1+q_3}c_1 + \frac{q_1}{q_1+q_3}c_3 < c_2,$$

but then it implies $v_2 > p^*$ because $v_2 > c_2$, that is, type (c_2, v_2) consumer will also buy insurance, which is a contradiction.

Now we consider the role of loading factors, which include both underwriting-based loading and claim-based loading. Denote the loading factor by $\ell > 0$, and by $p^*(\ell)$ the competitive equilibrium price in a market with the loading factor ℓ, which is determined by:

$$p^*(\ell) = (1+\ell)x\mathbb{E}[M|\mathcal{B}(p^*(\ell))]. \tag{10}$$

Define

$$m^\dagger \equiv \mathbb{E}(M) \tag{11}$$

as the *average risk type* in the population of consumers. We have the following result[17]:

Proposition 2 (Positive Correlation Property Holds in Competitive Equilibrium with Low Loadings). Suppose Assumptions 1, 2, and 3 are satisfied. A sufficient condition for the positive correlation property to hold in equilibrium if the insurance market is perfectly competitive is

$$\ell \leq \frac{v(m^\dagger, \underline{\lambda}; x)}{xm^\dagger} - 1. \tag{12}$$

Note that the upper bound on the loading factor specified by (12) depends on the ratio of the WTP relative to its expected claim for a consumer who has the average risk and the lower-bound risk preference. Proposition 2 shows that if the loading factor is bounded above by (12), then the competitive insurance market will always exhibit a positive correlation property in equilibrium, even in the presence of loading factors. The intuition is again quite simple. In order

[17] If the loading factor is additive instead of multiplicative, that is, if the equilibrium premium satisfies

$$p^*(\ell) = x\mathbb{E}[M|\mathcal{B}(p^*(\ell))] + \ell,$$

then the corresponding sufficient condition for Proposition 2 is

$$\ell \leq v\left(m^\dagger, \underline{\lambda}; x\right) - xm^\dagger.$$

for an equilibrium that the low-risk types purchase insurance, whereas the high-risk types do not, to exist, it must be the case that the premium is higher than the WTP for the high-risk types. However, the only way for such levels of premium to be consistent with equilibrium when the insured is actually of low-risk types is that the loading factor is very high, which is ruled out by the upper bound (12) on the loading factor.

Remark 4. We could have redefined consumers' "risk type" to be inclusive of the insurance loadings. Such a redefinition of risk type will make Assumption 3 more stringent. The sufficient condition (12) stated in Proposition 2 requires the risk premium for the average risk type to be sufficiently high.

To illustrate why the stated upper bound on loading factors is sufficient to rule out the negative correlation property in a competitive equilibrium, we introduce loadings into Example 3.

Example 3 (Continued, Impossibility of Negative Correlation Property with Low Loadings). Suppose that $c_2 = \sum_{j\in\{1,2,3\}} q_j c_j = \mathbb{E}[M]$; in words, c_2 is the average risk type $m^\dagger$ in the population of consumers, as defined in (11). Now, suppose that the market equilibrium price p^* is such that only type (c_1, v_1) consumers are purchasing insurance, that is, that the market equilibrium exhibits negative correlation property. For this to be an equilibrium, it must be the case that the equilibrium price $p^* = (1 + \ell)c_1$ and it satisfies

$$\max\{v_2, v_3\} < p^* \leq v_1. \tag{13}$$

However, this inequality is ruled out by the upper bound on the loadings in (12), which for this example is reduced to

$$1 + \ell \leq \frac{v_2}{c_2}. \tag{14}$$

To see this, note that (14) implies that

$$p^* = (1 + \ell)c_1 \leq \frac{v_2}{c_2} c_1 < v_2, \tag{15}$$

which is a contradiction against (13), implying that type (c_2, v_2) consumers would have preferred to purchase insurance as well at p^*.

Proposition 2 states that the positive correlation property is robust to a sufficiently small loading factor. However, when the loading factor is large enough, it is possible to obtain the negative correlation property. Again, we continue with Example 3 to elaborate this point.

Example 3 (Continued, Possibility of Negative Correlation Property with Moderate Loadings). Suppose for simplicity that $q_2 = 0$, and thus there are two types of consumers. In order for the market to exhibit the negative correlation property, we must have that type (c_1, v_1) consumers purchase the insurance policy, whereas type (c_3, v_3) consumers remain uninsured, that is,

$$v_3 \leq p^* = (1 + \ell)c_1 \leq v_1,$$

which is possible if and only if

$$\frac{v_3}{c_1} - 1 \leq \ell \leq \frac{v_1}{c_1} - 1. \tag{16}$$

Therefore, if the load is higher than $\frac{v_3}{c_1} - 1$ but lower than $\frac{v_1}{c_1} - 1$, there exists an equilibrium in which negative correlation property holds. Note that (16) indicates that a necessary condition for negative correlation property to emerge is that $v_1 \geq v_3$, which cannot be satisfied with one-dimensional consumer heterogeneity in risk types.

The logic underlying Propositions 1 and 2 suggests that whether multidimensional private information, particularly private information related to risk preferences, can explain the observed negative correlation between insurance purchase, and *ex post* risk realization must be related to large loading factors, or some noncompetitive features of the insurance market.[18]

The importance of the size of the loading factor highlighted in Propositions 1 and 2 suggests that the different findings in Chiappori and Salanié (2000) and Cohen (2005) we mentioned in the Introduction can potentially result from the differential loading factors in the two markets. Recall that Cohen's (2005) data is from an online Israeli insurer, whereas Chiappori and Salanié (2000) is from a traditional French insurance company. The online insurer is likely to have a much lower loading than the traditional insurer. Therefore, Proposition 2 suggests that positive correlation property is more likely to hold in the Israeli data. Systematic empirical studies of the size of the loading factor are rare. de Meza and Webb (2001) note that "Between 1985 and 1995 for U.K. insurers, expenses as a percentage of premium income averaged 25% for motor insurance and 37% for property damage insurance." The Affordable Care Act in the United States regulates that health insurers need to maintain a minimum "medical loss ratio" — the fraction of premium that need to be used to pay for claims — of 80%, which implies a loading factor of no more than 25% [as $80\% = 1/(1 + 25\%)$].[19] It is useful to point out that such levels of *average* expense/premium ratio can still be consistent with the sufficient condition stipulated in Proposition 2, which is a condition imposed only on the average risk type $m^\dagger$ as defined by (12).

□ **Relationship to CJSS (2006).** Let us now discuss in details the connections between our Propositions 1–2 and the results in CJSS (2006). Formally, using the notation in CJSS, a contract C_i reimburses the insured an amount $R_i(L)$ when loss L occurs, and they say that contract C_2 covers more than contract C_1 if $R_2(L) - R_1(L)$ is nondecreasing in L. Let

$$\pi(C_i) = p_i - \int R_i(L)dF_i(L) - \Gamma$$

denote the per-contract profit from contract C_i, where $F_i(L)$ is the distribution of loss among consumers purchasing contract C_i, and Γ is the fixed costs associated with the contract, and it is assumed to be the same across the contracts. The *nonincreasing profit (NIP) condition* states that $\pi(C_2) \leq \pi(C_1)$ if contract C_2 covers more than C_1. CJSS's main result, Proposition 2, states that under assumptions on consumer rationality (which we also assume) and the nonincreasing profit condition, it must be true that

$$\int R_2(L)dF_2(L) \geq \int R_2(L)dF_1(L). \tag{17}$$

In the notation of our article, the case we considered in Proposition 1 is the case with $\Gamma = 0$, $R_2(L) = xL$, and $R_1(L) = 0$, where $L = M$.[20] The NIP condition is automatically satisfied in our competitive market setting because of the zero-profit condition. Under this interpretation, CJSS's inequality (17) is equivalent to

$$\mathbb{E}[M|\mathcal{B}(p^*)] \geq \mathbb{E}[M|\mathcal{N}B(p^*)].$$

Thus, our result as stated in Proposition 1 is consistent with CJSS's inequality (17).

[18] This contrasts with a view that was shared by many in the literature. For example, Chetty and Finkelstein (2013) stated that "... if preferences are sufficiently important determinants of demand for insurance and sufficiently negatively correlated with risk type, the market can exhibit what has come to be called advantageous selection." See, however, CJSS (2006).

[19] See www.healthcare.gov/glossary/medical-loss-ratio-MLR/.

[20] When $\Gamma > 0$, CJSS's Proposition 1 cannot be used to show that the average *ex post* realization of risk for the insured is higher than the uninsured, as CJSS pointed out in their footnote 5.

When there is a proportional loading factor $\ell > 0$, the per-contract profit from contract C_i is then

$$\pi(C_i) = p_i - (1+\ell)\int R_i(L)dF_i(L),$$

and CJSS show that their testable implication is given by (their inequality (7), with tax rate $t = 0$)[21]:

$$\int R_2(L)dF_2(L) - \int R_2(L)dF_1(L) \geq \ell\left[\int R_1(L)dF_1(L) - \int R_2(L)dF_2(L)\right]. \quad (18)$$

Again, if we let $R_2(L) = xL$ and $R_1(L) = 0$ where $L = M$ to match the setting considered in our Proposition 2, the inequality (18) can be simplified as

$$\mathbb{E}[M|\mathcal{B}(p)] - \mathbb{E}[M|\mathcal{N}B(p)] \geq -\ell\mathbb{E}[M|\mathcal{B}(p)],$$

which does not correspond to the positive correlation property, even if the loading factor ℓ is sufficiently small. Indeed, CJSS comment that, "... we can test some well-defined implication of asymmetric information (*which may not look like a positive correlation property any more*)" [italics added]. We thus believe that our Proposition 2 is complementary to CJSS (2006) and provides some new insights to the existing literature.

5. Monopolistic insurance market

■ In this section, we focus on the other extreme of the insurance market structure, assuming that there is a monopolistic insurance firm that chooses a premium to maximize its profit.[22] We ask whether correlated multidimensional private information can lead to the emergence of negative correlation property (see Definition 1) in a monopolistic insurance market.[23]

We first provide some background on how we will model dependence of the two dimensional private information M and Λ. In Section 3, we stated that, in the population, consumers' type, (m, λ), is independently drawn from joint CDF $H(m, \lambda)$, with marginal CDFs for M and Λ, respectively, denoted by $F(\cdot)$ and $G(\cdot)$. It turns out to be easier to parameterize the dependence structure of the two random variables M and Λ using the concept of *copula*.[24] By Sklar's Theorem, for every joint distribution $H(m, \lambda)$, there exists a unique copula $\mathcal{C}(\cdot, \cdot)$ such that $H(m, \lambda) = \mathcal{C}(F(m), G(\lambda))$. That is, the dependence structure between M and Λ can be represented by a copula and remains unchanged under strictly increasing transformations of the random variables.

We first consider the case of *positive dependence* between the risk type M and the risk preference type Λ. Although intuition suggests that the positive correlation between risk and risk preference would exacerbate adverse selection and thus strengthen the positive correlation between insurance coverage and *ex post* realization of risk, here we provide a precise sufficient condition for such a conclusion.

Definition 2 (Positive Stochastic Monotonicity Dependence). Λ is stochastically increasing in M if $\Pr(\Lambda > \lambda | M = m)$ is a nondecreasing function of m for all λ.

Nelsen (2006) proved that Definition 2 is equivalent to $\mathcal{C}_{11}(z_1, z_2) \leq 0$ for all $(z_1, z_2) \in [0, 1]^2$ in the language of copula. Positive stochastic dependence means that a high realization of z_1 shifts the conditional distribution of z_2 according to first-order stochastic dominance. Because marginal

[21] CJSS used the notation $\mathbb{E}_i[L]$ to denote the expected claims under contract C_i, and they wrote $\mathbb{E}_i[L] = \int LdF_i(L)$. We believe that it is a typo and should be $\mathbb{E}_i[L] = \int R_i(L)dF_i(L)$.

[22] We will allow the monopolistic insurance firm to choose both the premium and the coverage in Section 7.

[23] In the online web Appendix B, we introduce a parameterization of the imperfectly competitive market structure and show that our results for the monopoly case are robust.

[24] See Nelsen (2006) for an excellent introduction to copulas.

distribution functions are monotonic, this property of copula translates directly into corresponding dependence property of the joint distribution of (M, Λ) .

Proposition 3 (Positive Stochastic Monotonicity Dependence Implies Positive Correlation Property). Suppose Assumptions 1, 2, and 3 are satisfied. If Λ is stochastically increasing in M, then positive correlation property holds under monopoly.

In fact, the positive correlation property result in Proposition 3 applies to any market structure. Moreover, it is useful to point out that Proposition 3 is general in the sense that it does not rely on the functional form of consumer's WTP. As long as consumers' WTP is increasing in both m and λ, Proposition 3 holds.

In the rest of the section, we focus on the case in which the risk type M and the risk preference type Λ exhibit *negative* dependence (to be made precise below) and investigate whether the negative dependence between M and Λ may lead to the emergence of negative correlation property under a monopolistic market structure. We consider the tractable case of CARA utility function and normally distributed shocks as described in Example 2. Note that for this CARA-Normal specification, it is without loss of generality to assume that $x = 1$[25]; thus from (4), we have $C(\theta) = m$, and

$$v(m, \lambda) = m + k\lambda, \quad \text{where } k \equiv \frac{\sigma^2}{2}. \tag{19}$$

We will interpret the parameter $k \equiv \sigma^2/2$, where σ^2 is the variance of the health expenditure shock, as the *relative importance of risk aversion* as a determinant of the consumer's WTP for insurance: a higher σ^2 means that consumers are subject to more volatility in health expenditure, and as a result, risk aversion becomes more important in determining the WTP for insurance.

□ **The role of preferences.** We study the effect of the relative importance between risks and preferences, that is, the magnitude of k defined in (19), holding fixed the joint distribution $H(\cdot, \cdot)$.

Proposition 4. Suppose that consumers have CARA utility functions and experience normally distributed risks, as described in Example 2. For every $H(\cdot,)$, there exists a threshold $k^{\dagger} > 0$ such that for all $k < k^{\dagger}$, $\mathbb{E}[M|\mathcal{B}(p)] > \mathbb{E}[M]$ for all $p \in (\underline{m} + k\underline{\lambda}, \overline{m} + k\overline{\lambda})$.

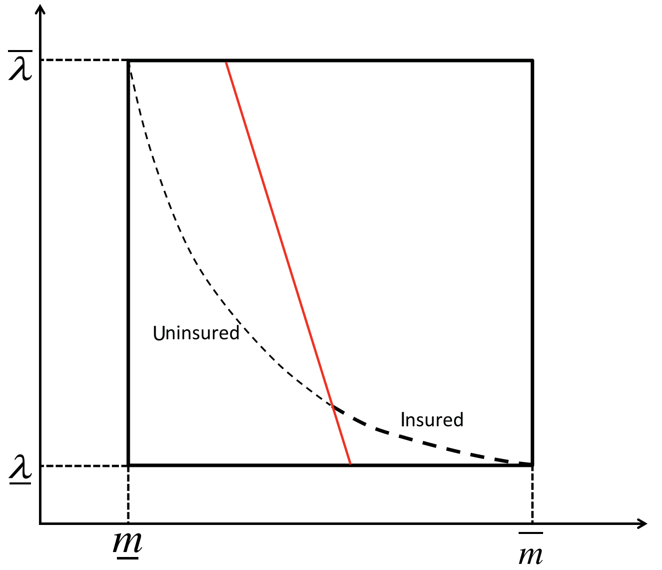
Proposition 4 shows that if risk preference is not a sufficiently important determinant of the demand for insurance, then negative correlation property will not emerge under monopolistic market structure, regardless of the joint distribution, $H(\cdot, \cdot)$, of risks and risk preferences. Notice that this holds true even when the risk type M and the risk preference type Λ exhibit strong negative dependence. Figure 1 illustrates why this is so for the extreme case when M and Λ exhibit perfect negative dependence. Because M and Λ are perfectly negative dependent, there exists a one-to-one monotonic mapping between m and λ, which is shown in the dashed line in Figure 1. A sufficiently small k yields a steep iso-WTP curve, which would imply that for any price in $(\underline{m} + k\underline{\lambda}, \overline{m} + k\overline{\lambda})$, the iso-WTP curve that separates purchasers and nonpurchasers of insurance at that price would intersect the dashed line as depicted: the higher risks (the darker segment of the dashed line) always purchase insurance before the lower risks (the lighter segment of the dashed line). This results in the positive correlation property.

Intuitively, when k is sufficiently small, a change in the price charged by the monopolist will have a stronger influence on the support of risk type M rather than that of Λ; as a result, the market is more susceptible to the risk-based adverse selection problem, as in the case of one-dimensional private information in risk, and the potential countervailing effect of selection based on risk preferences is too weak to override the positive correlation property.

[25] For $x \in (0, 1)$, we can redefine $\hat{v}((m, \lambda); 1) \equiv v((m, \lambda); x)/x$ and $\hat{C}(\theta; 1) \equiv C(\theta; x)/x$.

FIGURE 1

SELECTION BASED ON RISK TYPE: THE CASE OF LOW k [Color figure can be viewed at wileyonlinelibrary.com]



Now we consider the other limiting case, and show that if risk preference is a sufficiently important determinant for the demand of insurance, then when risk M and risk preference Λ are negatively dependent (to be defined more precisely below), a monopolistic market may exhibit negative correlation property in equilibrium. To this end, we first introduce the notion of *negative quadrant dependence*:

Definition 3 (Strict Negative Quadrant Dependence). M and Λ are strictly negatively quadrant dependent if for all $(m, \lambda) \in [\underline{m}, \overline{m}] \times [\underline{\lambda}, \overline{\lambda}]$,

$$H(m, \lambda) < F(m)G(\lambda).$$

Strict negative quadrant dependence formalizes the notion that two random variables are negatively dependent if greater values of M are more likely to appear with smaller values of Λ and vice versa. In the language of copula, it is equivalent to $\mathcal{C}(z_1, z_2) < z_1 z_2$. Notice that independent random variables does not satisfy strict negative quadrant dependence (see Nelsen, 2006).

Proposition 5. Suppose that consumers have CARA utility functions and experience normally distributed risks as described in Example 2. If M and Λ are strictly negatively quadrant dependent, then there exists a threshold $k^{\dagger\dagger}$ such that the negative correlation property emerges under monopoly when $k > k^{\dagger\dagger}$.

The intuition for Proposition 5 is as follows. When risk aversion is a sufficiently important determinant of the demand for insurance, the iso-WTP curve is sufficiently flat in the (m, λ) space. Thus, a change in the price by the monopolistic firm will have a stronger impact on the distribution of the risk-aversion type than the distribution of risk types among the set of the purchasers; that is, the selection of the consumers are more based on risk-aversion type λ than on risk type m. In the limit when k is sufficiently large, it is profit maximizing for the monopolist to price in a way to select *only* consumers whose risk aversion is above $\lambda^* \equiv \arg\max \lambda[1 - G(\lambda)]$.

Because higher risk aversion is associated with lower risk type by the assumption of quadrant negative dependence, consumers who purchase insurance have lower average risk than the entire population.

Proposition 5 represents a striking difference from the result reported in Proposition 1 for the case of perfectly competitive market, where we show that negative correlation property will not emerge under any joint distribution of M and Λ. To better explain the intuition why market structure plays such an important role in whether or not negative dependence between M and Λ can lead to negative correlation property in equilibrium, it is useful to further examine the difference, when k gets large, between the competitive equilibrium price $p^*(k)$ and the monopolistic price $p^m(k)$.[26] In Lemma 3 of the Appendix, we show that under monopoly, $p^m(k)/k$ converges to λ^* when k gets large. Moreover, it is straightforward to show that $p^*(k) \leq \mathbb{E}[M] + k\underline{\lambda}$ when k is large enough.[27]

The difference between competitive and monopolistic market structure can be understood as follows. As k gets larger, it becomes less costly for the insurance firms to offer insurance due to the increase in the WTP for insurance for consumers of all risk types. This is true for both the competitive and the monopolistic market structure. However, in a perfectly competitive market, the competitive pressure will force insurance companies to reduce prices and cover more consumers, leading to a low price level in equilibrium. Specifically, the competitive equilibrium price is lower than $\mathbb{E}[M] + k\underline{\lambda}$ when k is sufficiently large. As a result, only consumers with low risk (lower than the unconditional expectation) and low degree of risk aversion choose to opt out (the lower dashed line in Figure 2). This implies directly that consumers with no insurance have lower average risk than the entire population.[28] In contrast, a monopolistic firm recognizes that, when M and Λ exhibit strong negative dependence, most densities concentrate on the diagonal, as indicated in Figure 2. To maximize profit, a monopolist will choose a higher price relative to the competitive equilibrium price p^* so as to exclude the higher risk consumers (the upper solid line in Figure 2).

□ **Comparative statics with respect to the degree of negative dependence.** Proposition 5 is a *limiting result* under monopoly, as the role of preference as a determinant of the demand for insurance becomes sufficiently important. In this subsection, we further parameterize the nature of the negative dependence between M and Λ, and examine its impact on the equilibrium outcomes of the monopolistic market, including premium, market size, and the correlation between insurance purchase and *ex post* realization of risk.

To this end, we assume that both M and Λ are uniformly distributed between 0 and 1, and the joint distribution of M and Λ can be represented by a Bivariate Fréchet copula parameterized by $\mu \in [0, 1]$:

$$\mathcal{C}(z_1, z_2; \mu) = \mu\mathcal{W}(z_1, z_2) + (1 - \mu)\Pi(z_1, z_2), \tag{20}$$

where $\Pi(z_1, z_2) \equiv z_1 z_2$ is the product copula that exhibits *independence* and $\mathcal{W}(z_1, z_2) \equiv \max\{z_1 + z_2 - 1, 0\}$ is the Fréchet lower bound that shows *perfect negative dependence* between z_1 and z_2. Hence, the parameter μ measures the *degree of negative dependence* between z_1 and z_2. With a slight abuse of notation, we use $p^m(\mu)$ to denote the monopolist's profit-maximizing

[26] We use $p^*(k)$ and $p^m(k)$ to indicate the competitive equilibrium price and the profit-maximizing price, respectively, when the relative importance parameter of the risk preference in WTP is k. $p^m(k)$ is formally defined in the Appendix in (A1).

[27] To see this, suppose $k > \tilde{k}^* := [\overline{m} - \mathbb{E}[M]]/\underline{\lambda}$ and $p^*(k) > \mathbb{E}[M] + k\underline{\lambda}$. Then the firm's expected profit is

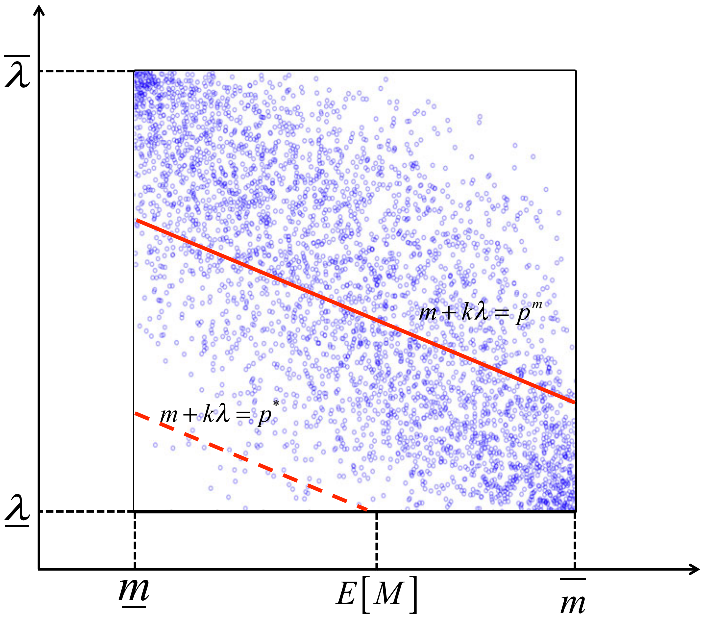
$$\pi(p^*(k); k) = \int_{\theta \in \mathcal{B}(p^*(k))} [p^*(k) - m] dH(m, \lambda) > \int_{\theta \in \mathcal{B}(p^*(k))} \left[\mathbb{E}[M] + \tilde{k}^*\underline{\lambda} - \overline{m}\right] dH(m, \lambda) = 0,$$

violating the zero-profit condition required for the competitive equilibrium.

[28] In the extreme case of perfect negative dependence, every consumer will buy insurance for any $k > 0$, hence, $\mathbb{E}[M|\mathcal{B}(p^*(k))] = \mathbb{E}[M]$.

FIGURE 2

PERFECT COMPETITION VERSUS MONOPOLY: THE CASE OF HIGH k [Color figure can be viewed at wileyonlinelibrary.com]



premium, and use $D(p^m(\mu))$ to denote the measure of consumers who will purchase insurance at the monopolist's price $p^m(\mu)$. We know from Proposition 1 that positive correlation property emerges for all k and μ if the market structure is perfect competition. For the case of monopolistic market structure, we have the following result:

Proposition 6. Suppose that consumers have CARA utility functions and experience normally distributed risks as described in Example 2, and the joint distribution is $H(m, \lambda; \mu) = \mu\mathcal{W}(m, \lambda) + (1 - \mu)\Pi(m, \lambda)$. Assume $M \sim \mathcal{U}[0, 1]$ and $\Lambda \sim \mathcal{U}[0, 1]$. Suppose the market structure is monopoly. Then,

1. If $k < 1$, then the positive correlation property emerges for all μ;
2. If $k > 1$, then there exists a threshold $\mu^\dagger$ such that:
   (a) the negative correlation property emerges if $\mu > \mu^\dagger$; and the positive correlation property emerges if $\mu < \mu^\dagger$;
   (b) $dp^m(\mu)/d\mu < 0$ and $dD(p^m(\mu))/d\mu > 0$.

The monopolist takes advantage of the increasing negative dependence between risk and risk aversion. In particular, when these two consumer characteristics become more negatively dependent, the monopolistic insurer has an incentive to set a high premium to rule out the "bad risks." As a result, the monopolist optimally chooses a price in the region where negative correlation property emerges.

6. Local adverse/advantageous selection and positive/negative correlation property

■ In this section, we use the example we analyzed in the previous section to clarify the connections between some important concepts related to the selection markets, such as adverse/advantageous selection and positive/negative correlation property. In the classic models of insurance with one-dimensional consumer heterogeneity in risk types (e.g., Arrow, 1963; Pauly, 1974; Rothschild and Stiglitz, 1976; Wilson, 1977), *adverse selection* refers to the idea that the average risk of those who choose to purchase insurance worsens as the insurance premium rises. The reason is very simple: as premium rises, the marginal consumer to drop out of coverage is the least risky among the insured in a *one*-dimensional model. To the extent that the average risks will translate into average cost of coverage for the insurance company, this translates into the average cost curve (and the marginal cost curve) being an increasing function of price.[29]

Einav, Finkelstein, and Cullen (2010) generalize this insight to models of potential multidimensional consumer heterogeneity. Their approach to test for the nature of selection in the insurance market treats the marginal cost curve as a *sufficient statistics* for the distribution of consumers' potentially multidimensional heterogeneity, and to the extent that researchers have access to exogenous price variations to estimate the marginal cost curve, it would indeed be a very attractive approach relative to a fully structural alternative.

The first clarification we would like to make in this section is the following: in models with multidimensional consumer heterogeneity, the average cost and the marginal cost curves are *typically nonmonotonic* functions of premium.[30] Thus, it is useful to define a *local* notion of selection based on how the marginal cost curve, which we will denote by $MC(p)$, is *locally* related to the premium change:

Definition 4 (Local Adverse/Advantageous Selection). The market is said to be subject to local adverse selection at price p if $MC'(p) > 0$. Similarly, the market is said to be subject to local advantageous selection at price p if $MC'(p) < 0$.

Note that local adverse/advantageous selection defined above is not an equilibrium notion and can be defined on any price p.

Remark 5. If consumers have *one*-dimensional heterogeneity in risk, local selection is *always* adverse, as the marginal buyer is always riskier as the premium increases; thus, there is no distinction in the notion of local selection and global selection. Moreover, in the one-dimensional heterogeneity model, the average cost curve, which we denote by $AC(p)$, is always above the marginal cost curve except at the price levels that exceed the maximum WTP for insurance (where the two curves coincide). Thus, $AC'(p) > 0$.

Remark 5 also implies that local advantageous selection is a phenomenon that may arise only when consumer heterogeneity is multidimensional.

Remark 6. When consumers have multidimensional heterogeneity, whether $AC(p)$ is increasing (or decreasing) at p depends on whether $AC(p)$ is larger (or smaller) than $MC(p)$. Thus, marginal cost curve locally increasing at $p$does not imply, and is not implied by the average cost locally increasing at p.

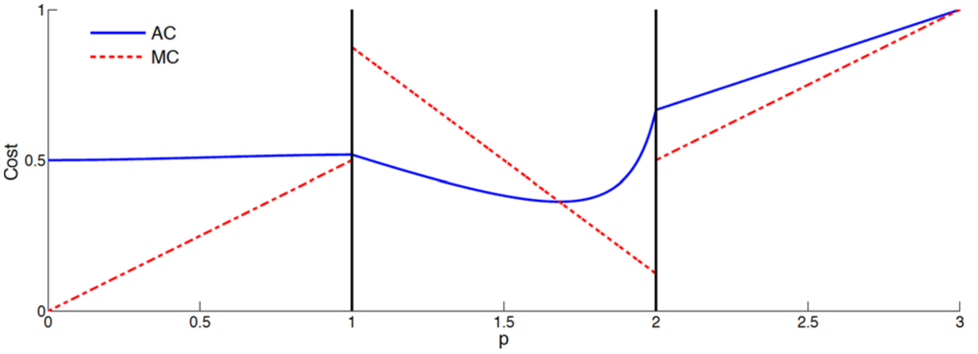
In contrast, the concept of *positive or negative correlation* property (see Definition 1 for the case of competitive market structure, but can be obviously generalized to any market structure)

[29] Equivalently, the average (and the marginal) cost curve is a decreasing function of the fraction insured, because the fraction insured is a monotonically decreasing function of the price.

[30] As we pointed out in footnote 7, Einav and Finkelstein (2011) note this as well in a footnote in their article, but proceed assuming monotonicity. Mahoney and Weyl (2017) also realize the importance of studying nonmonotone cost curves (footnote 4) : "It is possible that these slopes have different signs over different ranges or that the two have slopes of different signs over a particular range. All of these cases do not fall cleanly into one category or the other and are not our focus in what follows. It would be interesting to extend our analysis to such cases." Our discussion below is to emphasize that the nonmonotonicity is a typical property of models of multidimensional heterogeneity.

FIGURE 3

COST CURVES FOR $\mu \in (0, 1)$, $k = 2$ [Color figure can be viewed at wileyonlinelibrary.com]



refers to whether the *equilibrium* correlation between insurance purchase and *ex post* realization or risk is positive or negative. It is a property of the equilibrium, which depends on the *complete* distribution of consumer heterogeneity, whereas the local adverse/advantageous selection is determined by the *local* properties of the distribution of consumer heterogeneity on the iso-WTP subspace — recall that local selection can be defined on any price, including but not restricted to the equilibrium price. Of course, the equilibrium market price also depends on the market structure: in the price-cost graph as in Figure 3, the competitive market equilibrium price is determined by the intersection of the 45 degree line with $AC(p)$ curve, whereas the monopolistic equilibrium price is determined by the intersection of the marginal revenue curve, which is $p + D(p)/D'(p)$, with $MC(p)$ curve.

Now, we illustrate the notions of local adverse/advantageous selection in the context of the environment we studied in Section 5. We define, as a function of premium p, the demand for insurance, the total (expected) cost, the average (expected) cost, and the marginal (expected) cost at price p are equal to:

$$D(p) = \int_{\theta \in \mathcal{B}(p)} dH(m, \lambda),$$

$$TC(p) = \int_{\theta \in \mathcal{B}(p)} m dH(m, \lambda),$$

$$AC(p) = \mathbb{E}[M|\mathcal{B}(p)] = \frac{TC(p)}{D(p)},$$

$$MC(p) = \mathbb{E}[M|v(\theta) = p] = \frac{\int_{\theta \in \{\theta: v(\theta;1)=p\}} m dH(m, \lambda)}{\int_{\theta \in \{\theta: v(\theta;1)=p\}} dH(m, \lambda)} \equiv \frac{dTC(p)}{dp} \bigg/ \frac{dD(p)}{dp}.$$

Let us focus on the case where $k > 1$.

When $\mu = 0$, that is, when the risk type and risk preference type are independent and uniformly distributed, $MC(p; \mu = 0)$ is given by:

$$MC(p; \mu = 0) = \begin{cases} \frac{1}{2}p, & \text{if} \quad p \in [0, 1] \\ \frac{1}{2}, & \text{if} \quad p \in [1, k] \\ \frac{1}{2}[(p - k) + 1], & \text{if} \quad p \in [k, k + 1]. \end{cases}$$

It can be verified that $MC(p; \mu = 0)$ is nondecreasing in p.[31] Hence, the AC curve lies above the MC curve and is increasing in p for $p \in [0, k+1]$. This corresponds to the graphical representation of *adverse selection* in Einav, Finkelstein, and Cullen (2010) and Mahoney and Weyl (2017). It is worth noting that the monotonicity of MC and AC in this example is a sufficient but not a necessary condition for the emergence of positive correlation property under any market structure. Indeed, from Proposition 3, we must have $AC(p) > AC(\underline{m} + k\underline{\lambda}) = \mathbb{E}[M]$ for $p \in (\underline{m} + k\underline{\lambda}, \overline{m} + k\overline{\lambda})$ when $\mu = 0$, regardless the shape of cost curves.

When $\mu = 1$, that is, when M and Λ are perfectly negative dependent, $MC(p; \mu = 1)$ is given by

$$MC(p; \mu = 1) = \frac{k-p}{k-1} \quad \text{for } p \in [1, k].$$

The joint distribution $H(\cdot, \cdot)$ degenerates to a one-dimensional distribution and there exists a one-to-one mapping between m and λ . In particular, $\lambda = 1 - m$. Consumer's WTP is $k + (1-k)m$, which is decreasing in m when $k > 1$. Therefore, the lower risk types have higher WTP for a given price. This implies directly that the risk of the marginal consumer is decreasing as price increases and $AC(p)$ is decreasing in p. This corresponds to the graphical representation of *advantageous selection* in Einav, Finkelstein, and Cullen (2010) and Mahoney and Weyl (2017).

When $\mu \in (0, 1)$, the marginal cost $MC(p; \mu \in (0, 1))$ curve is given by:

$$MC(p; \mu \in (0,1)) = \begin{cases} \frac{1}{2}p & \text{if} \quad p \in [0,1] \\ \left[\frac{\mu}{k-1}\frac{k-p}{k-1} + \frac{1-\mu}{2k}\right] \Big/ \left[\frac{\mu}{k-1} + \frac{1-\mu}{k}\right] & \text{if} \quad p \in [1,k] \\ \frac{1}{2}[(p-k)+1] & \text{if} \quad p \in [k, k+1]. \end{cases}$$

It can be verified that MC is *nonmonotone* in p. As depicted in Figure 3, the MC curve is increasing in p for $p \in [0, 1]$ and $p \in [k, k+1]$, and decreasing for $p \in [1, k]$. Hence, in the notion of local selection as given by Definition 4, the local selection is adverse for $p \in [0, 1]$ and $p \in [k, k+1]$, but the local selection is advantageous when $p \in [1, k]$

The *nonmonotonicity* of the MC curve is not a unique property of this example. In fact, when we model dependence on the joint distribution with positive density almost everywhere on $[\underline{m}, \overline{m}] \times [\underline{\lambda}, \overline{\lambda}]$, it is impossible to obtain a globally decreasing MC curve. To see this, notice that only consumers with high risks purchase insurance when the price is below but sufficiently close to the maximum price $\overline{m} + k\overline{\lambda}$. Hence, the MC is high when price is close to $\overline{m}$. By the same token, when the price decreases gradually and is approaching to the minimum price $\underline{m} + k\underline{\lambda}$, the additional consumers are those with low risks. In other words, MC is low (close to $\underline{m}$) near the lowest price. Therefore, MC always starts up low and ends up high. For negative correlation property to emerge, MC needs to change its monotonicity *at least twice*, as Figure 3 illustrates.[32] Only under joint distribution functions that put zero density on certain combinations of (m, λ) is it possible to obtain a globally decreasing MC curve. For example, when M and Λ exhibit perfectly negative dependence (i.e., $\mu = 1$), the MC curve is globally decreasing as in Einav, Finkelstein, and Cullen (2010) and Mahoney and Weyl (2017).

Having clarified the distinction between the local notion of adverse/advantageous selection in models of multidimensional heterogeneity and the equilibrium (and thus, global) notion of positive/negative correlation property, the following claim discusses their connections:

[31] If $M$and Λ are not uniformly distributed, as in this example, the independence of M and Λ does not generally imply that the MC curve is monotonic in p. See Example A1 in the online web Appendix C for a construction.

[32] If M and Λ are supported on semiinfinite intervals, then the monotonicity of the MC curve needs to be changed at least once but not twice. To see this, suppose that $(m, \lambda) \in [\underline{m}, \infty) \times [\underline{\lambda}, \infty)$. In such a scenario, the maximum price can be arbitrarily large. Again, the marginal cost at the lowest price $\underline{m} + k\underline{\lambda}$ is $\underline{m}$; whereas, the marginal cost at any other price must be no less than $\underline{m}$ because $\underline{m}$ is the lowest risk type. Therefore, we must have that $\lim_{p \downarrow \underline{m}+k\underline{\lambda}} MC(p) = \underline{m}$ and $\lim_{p \uparrow \infty} MC(p) \geq \underline{m}$, which in turn indicates that MC needs to change its monotonicity *at least once* for negative correlation property to emerge.

Claim 1 (Connection between Local Advantageous/Adverse Selection and Equilibrium Negative/Positive Correlation Property).

1. If the market exhibits negative correlation property in equilibrium, then the market is subject to local advantageous selection at *some* prices.
2. However, the reverse does not hold.
3. Moreover, it is possible that selection is locally advantageous at the equilibrium price p^*, yet the market exhibits positive correlation property in equilibrium.

The first part of Claim 1 is obvious. Suppose that the market is subject to local adverse selection for all prices. Then, $MC(p)$ is increasing in p for all p by Definition 4. It follows immediately that $AC(p)$ is increasing in p globally, which implies that $\mathbb{E}[M|\mathcal{B}(p^*)] = AC(p^*) > AC(0) = \mathbb{E}[M]$. We provide an example to illustrate the second and third part of Claim 1.

Example 4. Suppose $m \in \{0.1, 0.2, 0.9\}$ with $\Pr(m = 0.1) = \Pr(m = 0.2) = \Pr(m = 0.9) = 1/3$, and $\lambda \in \{0.3, 0.7\}$ with $\Pr(\lambda = 0.1) = \Pr(\lambda = 0.3) = 1/2$. In addition, we assume that M and Λ are independent and $v(m, \lambda; x) = m + 2\lambda$. Consumer's willingness to pay is summarized as follows:

$\lambda \setminus m$	$m = 0.1$	$m = 0.2$	$m = 0.9$
$\lambda = 0.1$	$v = 0.3$	$v = 0.4$	$v = 1.1$
$\lambda = 0.3$	$v = 0.7$	$v = 0.8$	$v = 1.5$

The average cost and the demand curve can be derived as follows:

p	[0, 0.3]	(0.3, 0.4]	(0.4, 0.7]	(0.7, 0.8]	(0.8, 1.1]	(1.1, 1.5]
$AC(p)$	0.4	0.46	0.525	0.67	0.9	0.9
$D(p)$	1	5/6	4/6	3/6	2/6	1/6

It is straightforward to verify that there exist two competitive equilibrium premiums in this example: $p_1^* = 0.9 \in (0.8, 1.1]$ and $p_2^* = 0.525 \in (0.4, 0.7]$. From Proposition 1, the market always exhibits positive correlation property under both equilibria. For $p_1^* = 0.9$, the marginal cost increases from 0.2 to 0.9 as the premium increases from 0.8 to 1.1, indicating local adverse selection. [33] Interestingly, for $p_2^* = 0.525$, the marginal cost decreases from 0.2 to 0.1 as the premium increases from 0.4 to 0.7, indicating local advantageous selection. Therefore, from this example, we know that the market can still be subject to local advantageous selection at the equilibrium premium p_2^*, even though the positive correlation property holds.

□ **Discussions.** Clarifying the distinction between *local* selection (adverse or advantageous) and the *equilibrium* positive/negative correlation is important. As noted by de Meza and Webb (2017), this distinction is sometimes not clearly made. Einav, Finkelstein, and Levin (2010) state that, "contract j is adversely selected if the expected cost of insuring j's enrollees under contract j is greater than the expected cost of insuring the population I under contract j" and is advantageously selected otherwise. This is in fact referring to the equilibrium positive or negative correlation property. In contrast, Einav, Finkelstein, and Cullen (2010) state that "... the sign of the slope of the marginal cost curve tells us whether the resultant selection is adverse (or the marginal cost is increasing in price) or advantageous (if marginal cost is decreasing in price)."

[33] We caution readers that the MC curve in this example is not well defined at all premium levels because consumers have discrete types. See the online web Appendix C for a continuous-type version of Example 4.

In our terminology, this is a local definition of adverse or advantageous selection. Because the marginal cost curve is typically nonmonotonic in price, a property Einav, Finkelstein, and Cullen (2010) do not emphasize, but Einav and Finkelstein (2011) do acknowledge, the two notions may reach different conclusions under multiple consumer heterogeneity, as explicitly shown in our Claim 1 above. This is a point that was already made by de Meza and Webb (2017) in their two-type (bad and good risks) example, where they also argue that the local sign of the slope of the *average* cost with respect to *quantity,* which may not be monotonic, can be a useful measure of selection as well.

Azevedo and Gottlieb (2017) introduce a notion of intensive margin selection coefficient that measures the difference between the marginal changes of the premium and the marginal increase in the cost of insuring the consumers choosing a particular level of coverage, if they were to switch to a contract with an infinitesimally better coverage. Notice that their notion is with respect to the local changes in coverage, as opposed to the local changes in the premium used in our definition of local adverse/advantageous selection. They suggest that this notion is related to the positive correlation test. Indeed, if the intensive margin selection coefficient is positive (negative), it means that, *locally*, there is a positive (negative) correlation between the *ex post* risk realization and the insurance coverage generosity. More importantly, the intensive margin selection coefficient not only provides the sign of the correlation but also the magnitude of the positive correlation. They state that "It is possible that there is adverse selection in one region of the contract space, and advantageous selection in another region." In Section 7 below, we show in Proposition 7 that under a set of mild conditions, the multiple contract competitive equilibrium exhibits positive correlation property throughout the contract space. We will provide further connections between their intensive margin selection coefficient and the standard correlation test after we present Proposition 7.

7. Extension: endogenizing the contracts

■ In the basic model, the quality of the insurance is predetermined, and hence, we are comparing no purchase with purchasing $x \in (0, 1]$. In this section, we relax this assumption and show that the main results derived in both the competitive (Section 4) and monopolistic (Section 5) cases are indeed robust.

□ **Competitive insurance market: endogenous contract.** Consider a perfectly competitive market. Instead of allowing the competing insurance firms to choose the quality of the contract arbitrarily, we assume that a firm can provide contracts from a set $X = \{x_0, x_1, \cdots, x_N\}$ with $0 = x_0 < x_1 < \cdots < x_N \leq 1$, where x_0 refers to the null contract costing nothing (i.e., $C(\theta; x_0) \equiv x_0 \cdot m = 0$) and providing zero utility (i.e., $U(\theta; x_0, 0) = 0$) to all consumers. This approach allows us to endogenize the insurance quality with a minimal departure from the basic model we considered in Section 3. We would like to study whether negative correlation or positive correlation property will emerge in equilibrium when comparing x_i versus x_j for $i < j$, as well as those who do not purchase. Denote the price of contract x_i by p_i for $i \in \{0, 1, \cdots, N\}$. Fixing the premium vector $\boldsymbol{p} = (p_0, p_1, \cdots, p_N)$, denote the set and measure of consumers that selects contract x_i by $\mathcal{B}_i(\boldsymbol{p})$ and $D_i(\boldsymbol{p})$, respectively, that is,[34]

$$\mathcal{B}_i(\boldsymbol{p}) \equiv \left\{\theta : U(\theta; x_i, p_i) \geq \max_{j<i}\{U(\theta; x_j, p_j)\}, U(\theta; x_i, p_i) > \max_{j>i}\{U(\theta; x_j, p_j)\}\right\}, \tag{21}$$

and

$$D_i(\boldsymbol{p}) \equiv \int_{\theta \in \mathcal{B}_i(\boldsymbol{p})} dH(m, \lambda). \tag{22}$$

[34] It is assumed that whenever a consumer is indifferent between two contracts, she always selects the one with the higher coverage.

The average *ex post* realization of risk among those who purchase insurance contract x_i, conditioning on $\mathcal{B}_i(\boldsymbol{p})$ being nonempty, is:

$$\mathbb{E}[M|\mathcal{B}_i(\boldsymbol{p})] = \frac{\int_{\theta\in\mathcal{B}_i(\boldsymbol{p})} m dH(m,\lambda)}{\int_{\theta\in\mathcal{B}_i(\boldsymbol{p})} dH(m,\lambda)}. \tag{23}$$

Next, we define the employed equilibrium notion, which borrows from Azevedo and Gottlieb (2017). This definition corresponds to the *weak equilibrium* in Azevedo and Gottlieb (2017).

Definition 5. The price vector $\boldsymbol{p}^* = (p_0^*, p_1^*, \ldots, p_N^*)$ is a competitive equilibrium if

i. For each contract x_i, firms make no profits;
ii. Consumers make purchase decision and choose contracts optimally.

This price-taking definition requires firms to earn zero profit on each contract (either with positive or zero demand) and hence, rules out cross-subsidies between contracts.[35] The existence of equilibrium is guaranteed by Theorem 1 and Proposition 1 in Azevedo and Gottlieb (2017). It is worth noting that the equilibrium premium of the null contract is zero if the corresponding demand is strictly positive.

Next, we generalize the concept of positive and negative correlation property in Definition 1 to the current case with multiple contracts. It is useful to denote the set of contracts that induces positive insurance demand by $\tilde{X}(\boldsymbol{p})$ and the corresponding price vector by $\tilde{\boldsymbol{p}}$, holding fixed $\boldsymbol{p}$ and X.

Definition 6 (Positive and Negative Correlation Property with Multiple Contracts). Suppose $|\tilde{X}(\boldsymbol{p}^*)| \geq 2$. The insurance market exhibits *positive correlation property*in equilibrium if for every pair $x_i, x_j \in \tilde{X}(\boldsymbol{p}^*)$ with $i > j$, $\mathbb{E}[M|\mathcal{B}_i(\boldsymbol{p}^*)] > \mathbb{E}[M|\mathcal{B}_j(\boldsymbol{p}^*)]$, and it exhibits *negative correlation property*if $\mathbb{E}[M|\mathcal{B}_i(\boldsymbol{p}^*)] < \mathbb{E}[M|\mathcal{B}_j(\boldsymbol{p}^*)]$.

We assume that the insurance coverage x affects consumers' WTP in the natural way. Specifically, we impose the following assumptions on the utility function $U(\cdot)$:

Assumption 4. Suppose $x_i > x_j$. Then, $U(\theta; x_i, p_j + (x_i - x_j)m) > U(\theta; x_j, p_j)$ for all p_j.

Assumption 4 is intuitive: if the difference in the premium between two contracts is equal to that in the expected cost, then consumers prefer the one with better coverage. It is useful to point out that Assumption 4 is a natural extension of Assumption 3 (for the single contract case), and it implies Assumption 3.[36]

Consider a pair of contracts (x_i, p_i) and (x_j, p_j) with $x_i \neq x_j$. The set of consumer types that are indifferent between the two contracts, namely, $\{\theta : U(\theta; x_i, p_i) = U(\theta; x_j, p_j)\}$, can be represented by an *indifference curve*, which we denote by $\lambda = \mathcal{I}_{ij}(m)$ for $i \neq j$. It is obvious that $\mathcal{I}_{ij}(m) = \mathcal{I}_{ji}(m)$. The following assumption imposes some additional properties of the indifference curves.

[35] As argued by Azevedo and Gottlieb (2017), this equilibrium concept can be justified as the limit of a strategic model with differentiated products. The intuition is simple: a firm that cross-subsidizes contracts has incentives to sell contracts with positive profits only.

[36] To see this, we first let $p_j = v(\theta; x_j)$. It follows from the inequality in Assumption 4 that

$$U(\theta; x_i, v(\theta; x_j) + (x_i - x_j)m) > U(\theta; x_j, v(\theta; x_j)) = U(\theta; 0, 0) = U(\theta; x_i, v(\theta; x_i)),$$

where the two equalities follow from the definition of $v(\cdot)$. The above condition together with $\partial U/\partial p < 0$ implies that $v(\theta; x_i) - v(\theta; x_j) > (x_i - x_j)m$. Next, we let $x_j = 0$ and the derived condition is reduced to $v(\theta; x_i) > x_i \cdot m$ from $v(\theta; 0) = 0$. This coincides exactly with the condition specified in Assumption 3.

Assumption 5. Suppose $x_i > x_j$. The indifference curve $\lambda = \mathcal{I}_{ij}(m)$ satisfies the following properties: (i) $\mathcal{I}_{ij}(m)$ is strictly decreasing in m; (ii) if $(\dot{m}, \dot{\lambda}) > (m, \mathcal{I}_{ij}(m))$, then type-$(\dot{m}, \dot{\lambda})$ consumer prefers contract (x_i, p_i) over (x_j, p_i); (iii) $\mathcal{I}_{i0}(m)$ and $\mathcal{I}_{j0}(m)$ obey single-crossing condition and $\mathcal{I}_{i0}(m)$ is steeper than $\mathcal{I}_{j0}(m)$ at the intersection.

Parts (i) and (ii) of Assumption 5 are natural generalizations of Assumption 2, and part (iii) imposes a single-crossing condition on the set of indifference curves.[37]

Lemma 1. Suppose that Assumptions 1, 4, and 5 are satisfied, and that $x_i > x_j > x_0$, and $x_i, x_j, x_0 \in \tilde{X}(\boldsymbol{p}^*)$. Then, $p_i^*/x_i > p_j^*/x_j$.

Lemma 1 states that the unit price of the high-quality insurance must be higher than that of the low-quality insurance in a competitive equilibrium. The intuition is as follows. Consumers can be roughly classified into four groups based on their risk type (high risk versus low risk) and risk preference type (high risk aversion versus low risk aversion). Single-crossing condition of the indifference curves guarantees that consumers selecting the low-quality insurance must have both lower risk and lower risk aversion, and consumers of higher risk types (independent of risk preference type) will select into the high-quality insurance. Now suppose that the unit price of the high-quality insurance is lower relative to that of the low-quality insurance. Then, providing the low-quality insurance helps the insurance firms to maintain a high premium and to attract consumers of low-risk type (and lower risk aversion). This results in a net profit and contradicts to the zero-profit condition required by the definition of equilibrium. Therefore, the unit price of insurance has to be strictly increasing in the insurance quality in a competitive equilibrium.

Proposition 7 (Positive Correlation Property Holds in Competitive Equilibrium with Multiple Contracts). Suppose that Assumptions 1, 4, and 5 are satisfied, and that $|\tilde{X}(\boldsymbol{p}^*)| \geq 2$ and $x_0 \in \tilde{X}(\boldsymbol{p}^*)$. Then, positive correlation property always holds in a competitive equilibrium without loadings.[38]

It is useful to connect the result stated in Proposition 7 to the notion of the intensive margin selection coefficient introduced in Azevedo and Gottlieb (2017), which they denoted by $S_I(x)$. They defined $S_I(x)$ for continuous contract space, but it is easy to adapt it to our discrete contract space setting. Specially, the intensive margin selection coefficient evaluated at contract x_n, $n = 0, 1, 2, \ldots, N-1$ can be written as:

$$S_I(x_n) = \overbrace{\frac{p_{n+1}^* - p_n^*}{x_{n+1} - x_n}}^{\text{Marginal Premium Change}} - \overbrace{\frac{1}{\int_{\theta \in \mathcal{B}_n(\boldsymbol{p}^*)} dH(\theta)} \frac{\int_{\theta \in \mathcal{B}_n(\boldsymbol{p}^*)} [C(\theta; x_{n+1}) - C(\theta; x_n)] dH(\theta)}{x_{n+1} - x_n}}^{\text{Average Marginal Cost Change}}, \tag{24}$$

where the first term reflects the marginal premium change per-unit increase in coverage locally at contract x_n, and the second term reflects the average marginal cost increase of covering *those consumers who purchase contract* x_n, that is, those with $\theta \in \mathcal{B}_n(\boldsymbol{p}^*)$, if they

[37] We can analytically prove that both Assumptions 4 and 5 are satisfied in Examples 1 and 2, with the only exception of Part (iii) of Assumption 5 for Example 1. Simulation shows that this part is also satisfied, at least for CARA and CARA Bernoulli utility function in Example 1. See the online web Appendix A for the details of the proof.

[38] We conjecture that we can use arguments analogous to those in the proof of Proposition 2 to show that positive correlation property holds in competitive equilibrium with multiple contracts under a positive proportional loading factor as long as it is sufficiently low.

were to switch to contract x_{n+1}. In a competitive equilibrium, we have, for $i = n$ and $i = n + 1$,

$$p_i^* = \mathbb{E}\left[C(\theta; x_i)|\theta \in \mathcal{B}_i(\boldsymbol{p}^*)\right] = \frac{\int_{\theta\in\mathcal{B}_i(\boldsymbol{p}^*)} C(\theta; x_i)dH(\theta)}{\int_{\theta\in\mathcal{B}_i(\boldsymbol{p}^*)} dH(\theta)}.$$

Thus, we can rewrite (24) as

$$\begin{aligned}
S_I(x_n) &= \frac{1}{x_{n+1} - x_n}\left[\frac{\int_{\theta\in\mathcal{B}_{n+1}(\boldsymbol{p}^*)} C(\theta; x_{n+1})dH(\theta)}{\int_{\theta\in\mathcal{B}_{n+1}(\boldsymbol{p}^*)} dH(\theta)} - \frac{\int_{\theta\in\mathcal{B}_n(\boldsymbol{p}^*)} C(\theta; x_{n+1})dH(\theta)}{\int_{\theta\in\mathcal{B}_n(\boldsymbol{p}^*)} dH(\theta)}\right] \\
&= \frac{1}{x_{n+1} - x_n}\left[x_{n+1}\left(\frac{\int_{\theta\in\mathcal{B}_{n+1}(\boldsymbol{p}^*)} m dH(\theta)}{\int_{\theta\in\mathcal{B}_{n+1}(\boldsymbol{p}^*)} dH(\theta)} - \frac{\int_{\theta\in\mathcal{B}_n(\boldsymbol{p}^*)} m dH(\theta)}{\int_{\theta\in\mathcal{B}_n(\boldsymbol{p}^*)} dH(\theta)}\right)\right] \\
&= \frac{x_{n+1}}{x_{n+1} - x_n}\left\{\mathbb{E}\left[M|(m,\lambda) \in \mathcal{B}_{n+1}(\boldsymbol{p}^*)\right] - \mathbb{E}\left[M|(m,\lambda) \in \mathcal{B}_n(\boldsymbol{p}^*)\right]\right\},
\end{aligned}$$

where the second equality follows from our assumption that $C(\theta; x_i) = x_i \cdot m$. Proposition 7 thus implies that in our setting, the intensive margin selection coefficient $S_I(x_n)$ defined in Azevedo and Gottlieb (2017) is always positive. Indeed, the examples in Azevedo and Gottlieb (2017), where they find changing signs of $S_I(\cdot)$ at different levels of x feature heterogeneity in *ex post* moral hazard, which may cause a violation of our Assumptions 2 and 4.

□ **Monopolistic insurance market: endogenous contract.** Now we consider a monopolistic insurance firm that chooses premium p^m *and* insurance coverage x^m to maximize its expected profit[39]:

$$\max_{\{p,x\}} \pi(p, x) \equiv \int_{\theta\in\mathcal{B}(p,x)} (p - xm)dH(m, \lambda), \tag{25}$$

where $\mathcal{B}(p, x)$ is defined as

$$\mathcal{B}(p, x) \equiv \{\theta : v(\theta; x) - p \geq 0\}. \tag{26}$$

For the sake of tractability, we employ the expression of WTP derived from the CARA-Normal specification in Example 2, that is, $v(\theta; x) = xm + x(2 - x)k\lambda$. For this class of examples, we will use $\pi(p, x; k)$ and $\mathcal{B}(p, x; k)$ to indicate that both are related to the parameter $k \equiv \sigma^2/2$ as defined in (19), which measures the relative importance of risk aversion as a determinant of the consumer's WTP for insurance. The following lemma can be implied immediately from the linearity of consumers' WTP.

Lemma 2. Suppose $v(\theta; x) = xm + x(2 - x)k\lambda$, then $\pi(p, x; k) = x\pi(p/x, 1; (2 - x)k)$.

The proof follows directly from the fact that $\mathcal{B}(p, x; k) = \mathcal{B}(p/x, 1; (2 - x)k)$ and is omitted for brevity. Lemma 2 uncovers the trade-off between x and the degree of adverse selection in an intuitive way: holding fixed the per-unit price of the insurance (i.e., p/x), increasing insurance coverage x will directly increase the revenue received from each consumer that purchases insurance, at the cost of yielding steeper iso-WTP curves in the (m, λ) space, which indicates more severe adverse selection.

The next two propositions illustrate the role of preferences and report results that are parallel to those in Proposition 4 and 5.

[39] We follow Veiga and Weyl (2016) and assume that the monopolist offers a single contract instead of a menu of contracts. See Jullien, Salanié, and Salanié (2007) for the investigation of optimal menu of contracts in a two-outcome/two-type model of moral hazard with adverse selection on the consumer's risk aversion.

Proposition 8. Suppose that consumers have CARA utility functions and experience normally distributed risks as described in Example 2. For every $H(\cdot,)$, there exists a threshold $\hat{k}^{\dagger} > 0$ such that for all $k < \hat{k}^{\dagger}$, $\mathbb{E}[M|\mathcal{B}(p, x; k)] > \mathbb{E}[M]$ for all $p \in (\underline{m} + k\underline{\lambda}, \overline{m} + k\overline{\lambda})$ and $x \in (0, 1]$.

Proposition 9. Suppose that consumers have CARA utility functions and experience normally distributed risks as described in Example 2. If M and Λ are negatively quadrant dependent, then there exists a threshold $\hat{k}^{\dagger\dagger}$ such that the negative correlation property emerges under monopoly when $k > \hat{k}^{\dagger\dagger}$.

Propositions 8 and 9 show that the results in Propositions 4 and 5 are robust to the endogenization of insurance quality in a monopolistic market. Before we explain the results, it is useful to discuss the sources of advantageous selection. Because both the joint distribution of M and Λ and the shape of the iso-WTP curves will influence the firm's cost curves, there are *two sources of advantageous selection* in a model of multidimensional private information. The first source of advantageous selection comes from the joint distribution of M and Λ, as emphasized in Section 5. Intuitively, negative dependence between M and Λ favors the monopolist and decreases the firm's cost of providing insurance. The second source of advantageous selection is a result of the downward sloping nature of the iso-WTP curves. Fixing the risk type, consumers of a high risk-aversion type are more willing to purchase insurance relative to those of a low-risk aversion type. As a result, consumers with *low* risk type and *high* risk-aversion type will purchase insurance, mitigating adverse selection compared to a model of one-dimensional private information in risk where the low-risk type will opt not to purchase insurance.

Fixing the joint distribution of M and Λ, different from the model of exogenous insurance quality, the monopolist is able to choose insurance coverage x to change the composition of consumers and hence, mitigates adverse selection it faces. However, the impact of x in influencing selection is limited. Formally, the absolute value of the slope of the iso-WTP curve is given by $(2 - x)k$, which is bounded from below by k and above by $2k$. Therefore, the effect of k in determining selection takes over as k becomes sufficiently large (respectively, small) and the result in Proposition 4 (respectively, Proposition 5) remains.

Next, we report some numerical results to shed light on the role of dependence between M and Λ on the design of the optimal contract, especially on the correlation between insurance purchase and *ex post* realization of risk. To proceed, we assume that $M \sim \mathcal{U}[0, 1]$, $\Lambda \sim \mathcal{U}[0, 1]$, and the joint distribution is $H(m, \lambda; \mu) = \mu\mathcal{W}(m, \lambda) + (1 - \mu)\Pi(m, \lambda)$, as in Section 5. With slight abuse of notation, we denote the monopolist's optimal contract by $(p^m(\mu, k), x^m(\mu, k))$.

We will first briefly describe the algorithm of searching for the optimal contract in the numerical analysis.[40] We use $\hat{p}^m(\mu, k)$ to denote the monopolist's optimal premium fixing $x = 1$. We first completely solve for $\hat{p}^m(\mu, k)$ for any k.

Suppose $k > 1$, the profit function is[41]:

$$\pi(p, 1; k) = \begin{cases} p - \frac{1}{2} & \text{for} \quad p \in (-\infty, 0] \\ \left(p - \frac{1}{2}\right) - (1 - \mu)\frac{1}{3k}p^3 & \text{for} \quad p \in [0, 1] \\ (1 - \mu)\left[\left(p - \frac{1}{2}\right) + \frac{-3p^2+3p-1}{3k}\right] + \frac{1}{2}\mu\frac{k-p}{k-1}\left[2p - \frac{k-p}{k-1}\right] & \text{for} \quad p \in [1, k] \\ (1 - \mu)\left[\frac{(p-1)^3}{3k} - \frac{(p-1)^2}{2} + \frac{k^2}{6}\right] & \text{for} \quad p \in [k, k + 1] \\ 0 & \text{for} \quad p \in [k + 1, \infty). \end{cases}$$

[40] The program used in the numerical analysis is available from the authors upon request.

[41] See the online web Appendix D for details of the derivation of the profit function, the demand curve, and the cost curves.

From the proof of Proposition 6, the optimal premium is,

$$\hat{p}^m(\mu, k) = \frac{\mu \frac{k^2}{k-1} + (1-\mu)\left(k - \frac{1}{k}\right)}{\mu\left(2 + \frac{1}{k-1}\right) + 2(1-\mu)\left(1 - \frac{1}{k}\right)}.$$

Similarly, suppose $k = 1$, the profit function is:

$$\pi(p, 1; 1) = \begin{cases} p - \frac{1}{2} & \text{for} \quad p \in (-\infty, 0] \\ \left(p - \frac{1}{2}\right) - (1-\mu)\frac{1}{3}p^3 & \text{for} \quad p \in [0, 1] \\ (1-\mu)\left[\frac{(p-1)^3}{3} - \frac{(p-1)^2}{2} + \frac{1}{6}\right] & \text{for} \quad p \in (1, 2] \\ 0 & \text{for} \quad p \in [k+1, \infty). \end{cases}$$

For this special case, the profit function is discontinuous at $p = 1$. It can be verified that $\hat{p}^m(\mu, 1) = 1$ and the positive correlation property emerges in this case.

Last, suppose $k < 1$, the profit function is[42]:

$$\pi(p, 1; k) = \begin{cases} p - \frac{1}{2} & \text{for} \quad p \in (-\infty, 0] \\ \left(p - \frac{1}{2}\right) - (1-\mu)\frac{1}{3k}p^3 & \text{for} \quad p \in [0, k] \\ (1-\mu)\left[\frac{1}{6}k^2 - \frac{1}{2}(p-1)^2\right] + \mu\left(k - \frac{1}{2}\right)\frac{(p-1)^2}{(k-1)^2} & \text{for} \quad p \in [k, 1] \\ (1-\mu)\left[\frac{(p-1)^3}{3k} - \frac{(p-1)^2}{2} + \frac{k^2}{6}\right] & \text{for} \quad p \in [1, k+1] \\ 0 & \text{for} \quad p \in [k+1, \infty). \end{cases}$$

It can be verified that the profit is increasing for $p \in [0, k]$ and decreasing for $p \in [1, k+1]$. Therefore, $\hat{p}^m(\mu, k) \in [k, 1]$. Moreover, if $\mu \leq (1/k - 1)^2$, the profit is increasing in p for $p \in [k, 1]$, indicating $\hat{p}^m(\mu, k) = 1$. If $\mu > (1/k - 1)^2$, the profit is decreasing in p for $p \in [k, 1]$, indicating $\hat{p}^m(\mu, k) = k$. From Proposition 6, the positive correlation property emerges.

The optimal coverage $x^m(\mu, k)$ solves the following one-dimensional optimization problem:

$$\max_{x \in [0,1]} x \cdot \pi(\hat{p}^m(\mu, (2-x)k), 1; (2-x)k). \tag{27}$$

After we numerically compute $x^m(\mu, k)$, the optimal premium is given by $p^m(\mu, k) = x^m(\mu, k) \cdot \hat{p}^m(\mu, [2 - x^m(\mu, k)]k)$ from Lemma 2.

Figure 4 graphically illustrates our numerical results. The solid curve is the combination of (μ, k), for which the expected risk conditional on purchase under the optimal contract is equal to the unconditional expectation (i.e., the contour plot of $\mathbb{E}[M|\mathcal{B}(p^m(\mu, k), x^m(\mu, k))] = \mathbb{E}[M]$ in the (μ, k) space). [43] The region of (μ, k) to the right (respectively, to the left) of the solid curve depicts the combination of (μ, k) for which the negative correlation property (respectively, the positive correlation property) emerges under optimal contract. The first pattern to notice is that fixing the degree of negative dependence between M and Λ, the negative correlation property (respectively, the positive correlation property) emerges under the optimal contract when k is sufficiently large (respectively, small). This confirms the results in Propositions 8 and 9. Second, the result in Proposition 6 is robust to endogenous insurance quality. Specifically, holding fixed the degree of relative importance of risk aversion, negative correlation property is more likely to appear when M and Λ are sufficiently negative dependent.[44]

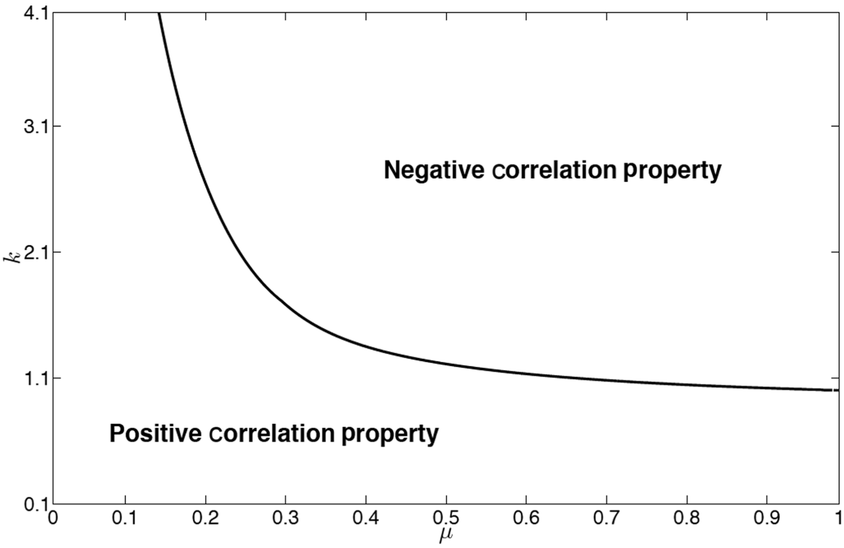
[42] See the online web Appendix E for details of the derivations for the case $k < 1$.

[43] The contour plots are shown only for $(\mu, k) \in [0, 1] \times [0.1, 4.1]$.

[44] Some readers may want to compare the contour plot of (μ, k) when $x = 1$ and that when x is endogenously chosen by the monopolist. Simulation shows that endogenizing x slightly shifts the contour plot to the left and enlarges the region of the negative correlation property under the optimal contract. This result is intuitive: the monopolist can better

FIGURE 4

POSITIVE OR NEGATIVE CORRELATION PROPERTY UNDER MONOPOLY WITH AN ENDOGENOUS CONTRACT



8. Conclusion

■ A large empirical literature has found that the correlation between insurance purchase and *ex post* realization of risk is often statistically insignificant or negative, which is inconsistent with the predictions from the classic models of insurance à la Akerlof (1970), Pauly (1974), and Rothschild and Stiglitz (1976), where consumers differ only in their risk types. It is suggested that the selection based on multidimensional private information, for example, risk type and risk preference type, may be able to reconcile the empirical findings. In this article, we investigate, under different market structures, whether selection based on multidimensional private information can result in negative correlation between insurance coverage and *ex post* realization of risk in equilibrium. We show that if the insurance market is perfectly competitive, selection based on multidimensional private information does *not* generate negative correlation property in equilibrium, unless there is a sufficiently high loading factor. If the insurance market is monopolistic, however, we show that it is possible to generate negative correlation property in equilibrium when risk type and risk preference type are sufficiently negative dependent, a notion we formalize using the concept of copula. We further show that this result generalizes when contracts are partially endogenized. We also clarify the confusions in this growing literature about the connections between some of the important concepts such as adverse/advantageous selection and positive/negative correlation property.

There are some interesting directions for future research. First, in this article, we studied the role of additional consumer heterogeneity in risk preference. It is important to model and examine whether other sources of heterogeneity, such as heterogeneity in moral hazard (Einav et al., 2013) and heterogeneity in imperfect rationality (e.g., Fang, Keane, and Silverman, 2008), will lead to different conclusions on the emergence of the positive or the negative correlation property. As we pointed out in footnote 12, introducing moral hazard is likely to lead to the violations of Assumption 2. Heterogeneity in imperfect rationality will call for a plausible behavioral model of consumers' insurance purchase decisions. Second, in our article, we have identified the potential

take advantage of the negative dependence of the joint distribution if it is allowed to design x. Therefore, the negative correlation property is more likely to emerge under optimal contract.

role of loading factors in possibly affecting the equilibrium of the insurance market under different market structures. In particular, Proposition 2 shows that a sufficiently small loading factor is sufficient to ensure that positive correlation property always holds under a competitive insurance market regardless of the dependence structure of the multidimensional heterogeneity. Loading factors can also drive a wedge between the WTP for insurance and marginal cost (inclusive of loading factors) of providing coverage. Further investigations, empirically about the magnitude of loading factors, and theoretically about how loading factors — potentially heterogeneous among insurance firms — may impact the equilibrium of the insurance market, are also an important avenue for future research. Finally, our article assumes that the contracts space is either exogenous (as in Sections 4 and 5) or endogenous in a restricted way (as in Section 7). Generalizing the analysis when insurers are allowed to choose menus of screening contracts is also an important area for future research.

Appendix

This Appendix contains the proofs omitted in the text.

□ **Proof of Proposition 1.**

Proof. We consider two cases depending on the level of the equilibrium premium relative to $v(m^{\dagger}, \underline{\lambda}; x)$, the WTP for insurance of type-$(m^{\dagger}, \underline{\lambda})$ consumer, where $m^{\dagger}$ is defined in (11).

Case I $p^* \leq v(m^{\dagger}, \underline{\lambda}; x)$. For any $\lambda = \tilde{\lambda} \in [\underline{\lambda}, \overline{\lambda}]$, we have

$$\begin{aligned}\mathbb{E}[M|(m, \tilde{\lambda}) \in \mathcal{N}B(p^*)] &= \mathbb{E}[M|v(m, \tilde{\lambda}; x) < p^*]\\ &\leq \mathbb{E}\left[M|v(m, \tilde{\lambda}; x) \leq v(m^{\dagger}, \underline{\lambda}; x)\right]\\ &\leq \mathbb{E}\left[M|v(m, \tilde{\lambda}; x) \leq v(m^{\dagger}, \tilde{\lambda}; x)\right]\\ &= \mathbb{E}\left[M|\Lambda = \tilde{\lambda}, M \leq m^{\dagger}\right] < m^{\dagger},\end{aligned}$$

where the first inequality follows from the assumption that $\partial v/\partial m>0$; the second inequality follows from the assumptions that $\partial v/\partial \lambda>0$ and $\partial v/\partial m > 0$. Therefore, the average risk conditional on no insurance purchase can be bounded from above by

$$\mathbb{E}[M|\mathcal{N}B(p^*)] = \int_{\underline{\lambda}}^{\overline{\lambda}} \mathbb{E}[M|(m, \tilde{\lambda}) \in \mathcal{N}B(p^*)]dG(\tilde{\lambda}|\mathcal{N}B(p^*)) < m^{\dagger} = \mathbb{E}[M].$$

From (7)–(9), it is clear that $\mathbb{E}[M|\mathcal{B}(p^*)] > \mathbb{E}[M|\mathcal{N}B(p^*)]$ follows from $\mathbb{E}[M|\mathcal{N}\mathcal{B}(p^*)] < \mathbb{E}[M]$.

Case II $p^* > v(m^{\dagger}, \underline{\lambda}; x)$. Because p^* is determined by (6) in a competitive insurance market without loadings, we have that

$$\mathbb{E}[M|\mathcal{B}(p^*)] = \frac{p^*}{x} > \frac{v\left(m^{\dagger}, \underline{\lambda}; x\right)}{x} > m^{\dagger} = \mathbb{E}[M],$$

where the second inequality follows from Assumption 3.

■

□ **Proof of Proposition 2.**

Proof. It is clear that the desired result still applies in Case I in the proof of Proposition 1. Under Case II, that is, if $p^*(\ell) > v(m^{\dagger}, \underline{\lambda}; x)$, then we have

$$\mathbb{E}[M|\mathcal{B}(p^*(\ell))] = \frac{p^*(\ell)}{(1+\ell)x} > \frac{v\left(m^{\dagger}, \underline{\lambda}; x\right)}{(1+\ell)x} \geq m^{\dagger} = \mathbb{E}[M],$$

where the second inequality follows from the postulated restriction (12) on the loading factor ℓ. ■

□ **Proof of Proposition 3.**

Proof. For the ease of exposition, we define the function $\tilde{\lambda}(m; p, x)$ so that $v(m, \tilde{\lambda}(m; p, x); x) = p$. In words, $(m, \tilde{\lambda}(m; p, x))$ is the point on the iso-WTP curve valued at p in the (m, λ) space; or equivalently, $\tilde{\lambda}(m; p, x)$ is the threshold risk preference type for risk type m who is indifferent between purchasing the insurance of quality x at premium p. Accounting for the lower and upper bounds of λ, we define

$$\hat{\tilde{\lambda}}(m; p, x) \equiv \min\left\{\max\left\{\tilde{\lambda}(m; p, x), \underline{\lambda}\right\}, \overline{\lambda}\right\}.$$

With slight abuse of notation, we drop x in $\tilde{\lambda}(\cdot)$ and $\hat{\tilde{\lambda}}(\cdot)$ in what follows. Assumption 2 implies that $\hat{\tilde{\lambda}}(m; p)$ is nonincreasing in m for all $p \in (v(\underline{m}, \underline{\lambda}; x), v(\overline{m}, \overline{\lambda}; x))$.

Fix any price $p \in (v(\underline{m}, \underline{\lambda}; x), v(\overline{m}, \overline{\lambda}; x))$. The marginal density of M conditional on purchasing insurance is

$$f(m|\mathcal{B}(p)) = \frac{\int_{\hat{\tilde{\lambda}}(m;p)}^{\overline{\lambda}} h(m,\lambda)d\lambda}{\int_{\underline{m}}^{\overline{m}} \int_{\hat{\tilde{\lambda}}(m;p)}^{\overline{\lambda}} h(m,\lambda)d\lambda dm}.$$

For $m'' > m'$, we must have:

$$\begin{aligned}
\frac{f(m''|\mathcal{B}(p))}{f(m'')} &= \frac{\int_{\hat{\tilde{\lambda}}(m'';p)}^{\overline{\lambda}} h(m'',\lambda)d\lambda}{\int_{\underline{m}}^{\overline{m}} \int_{\hat{\tilde{\lambda}}(m;p)}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \Bigg/ f(m'') \\
&\geq \frac{1}{\int_{\underline{m}}^{\overline{m}} \int_{\hat{\tilde{\lambda}}(m;p)}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \frac{\int_{\hat{\tilde{\lambda}}(m';p)}^{\overline{\lambda}} h(m'',\lambda)d\lambda}{f(m'')} = \frac{\Pr(\Lambda \geq \hat{\tilde{\lambda}}(m';p)|M=m'')}{\int_{\underline{m}}^{\overline{m}} \int_{\hat{\tilde{\lambda}}(m;p)}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \\
&\geq \frac{\Pr(\Lambda \geq \hat{\tilde{\lambda}}(m';p)|M=m')}{\int_{\underline{m}}^{\overline{m}} \int_{\hat{\tilde{\lambda}}(m;p)}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \\
&= \frac{\int_{\hat{\tilde{\lambda}}(m';p)}^{\overline{\lambda}} h(m',\lambda)d\lambda}{\int_{\underline{m}}^{\overline{m}} \int_{\hat{\tilde{\lambda}}(m';p)}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \Bigg/ f(m') = \frac{f(m'|\mathcal{B}(p))}{f(m')},
\end{aligned}$$

where the first inequality follows from the fact that $\hat{\tilde{\lambda}}(m; p)$ is nonincreasing in m for all p; and the second inequality follows from the definition of positive stochastic monotonicity dependence. Hence, $\frac{f(m|\mathcal{B}(p))}{f(m)}$ satisfies the monotone likelihood ratio property, which implies that $F(m|\mathcal{B}(p))$ first-order stochastically dominates $F(m)$. Moreover, Assumption 2 implies that the first-order stochastic dominance is strict for at least a positive measure of values of m. Therefore, $\mathbb{E}[M|\mathcal{B}(p)] > \mathbb{E}[M]$, which is equivalent to the positive correlation property as stated in Definition 1. ■

□ **Proof of Proposition 4.**

Proof. For notational convenience, denote the range of the risk preference type Λ by $R_\Lambda \equiv \overline{\lambda} - \underline{\lambda}$.

When $p \in (\underline{m} + k\underline{\lambda}, \mathbb{E}[M] + k\underline{\lambda})$, if a consumer's risk type m is such that $m \geq \mathbb{E}[M]$, then her WTP $v(m, \lambda) \geq m + k\underline{\lambda} \geq \mathbb{E}[M] + k\underline{\lambda} > p$, thus, such a consumer will for sure purchase insurance at price p. Thus, all the consumers who do not purchase insurance must have risk type $m < \mathbb{E}[M]$, which implies that $\mathbb{E}[M|\mathcal{N}B(p)] < \mathbb{E}[M]$, or equivalently, $\mathbb{E}[M|\mathcal{B}(p)] > \mathbb{E}[M|\mathcal{N}B(p)]$.

When $p \in (\mathbb{E}[M] + k\overline{\lambda}, \overline{m} + k\overline{\lambda})$, if a consumer's risk type m is such that $m \leq \mathbb{E}[M]$, then her WTP for insurance $v(m, \lambda) = m + k\lambda \leq \mathbb{E}[M] + k\overline{\lambda} < p$ for all $\lambda \in [\underline{\lambda}, \overline{\lambda}]$. Thus, the risk type of consumers who buy insurance at price p must be higher than $\mathbb{E}[M]$. This implies that $\mathbb{E}[M|\mathcal{B}(p)] > \mathbb{E}[M]$, or equivalently, $\mathbb{E}[M|\mathcal{B}(p)] > \mathbb{E}[M|\mathcal{N}B(p)]$.

Now, we consider the case when $p \in [\mathbb{E}[M] + k\underline{\lambda}, \mathbb{E}[M] + k\overline{\lambda}]$. Suppose $k < k_1$, where

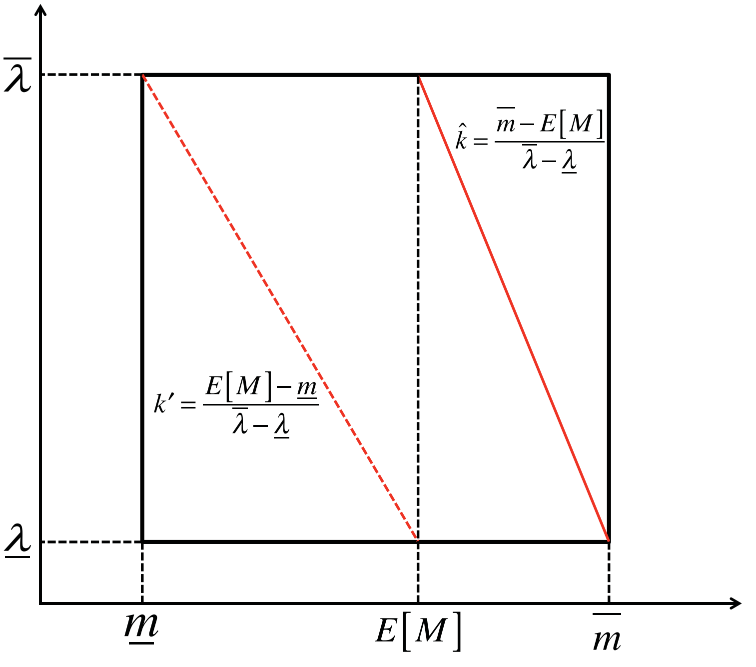
$$k_1 \equiv \min\left\{(\overline{m} - \mathbb{E}[M])/R_\Lambda, (\mathbb{E}[M] - \underline{m})/R_\Lambda\right\}.$$

Then, the iso-WTP curve of the marginal types of consumers, namely, those consumers with $v(m, \lambda) = m + k\lambda = p$, will intersect the upper and lower bounds of the range of Λ (see Figure A1). For $p \in [\mathbb{E}[M] + k\underline{\lambda}, \mathbb{E}[M] + k\overline{\lambda}]$, those and only those consumers to the right of the iso-WTP line will purchase insurance. Thus, we have :

$$\begin{aligned}
\mathbb{E}[M|\mathcal{N}B(p)] &= \frac{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\min\left\{\frac{p-m}{k}, \overline{\lambda}\right\}} m h(m,\lambda)d\lambda dm}{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\min\left\{\frac{p-m}{k}, \overline{\lambda}\right\}} h(m,\lambda)d\lambda dm} \\
&\leq \frac{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\min\left\{\frac{p-m}{k}, \overline{\lambda}\right\}} m h(m,\lambda)d\lambda dm + \int_{p-k\overline{\lambda}}^{p-k\underline{\lambda}} \int_{\frac{p-m}{k}}^{\overline{\lambda}} (p - k\underline{\lambda}) h(m,\lambda)d\lambda dm}{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\min\left\{\frac{p-m}{k}, \overline{\lambda}\right\}} h(m,\lambda)d\lambda dm + \int_{p-k\overline{\lambda}}^{p-k\underline{\lambda}} \int_{\frac{p-m}{k}}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \\
&= \frac{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\overline{\lambda}} m h(m,\lambda)d\lambda dm}{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} + \frac{\int_{p-k\overline{\lambda}}^{p-k\underline{\lambda}} \int_{\frac{p-m}{k}}^{\overline{\lambda}} (p - k\underline{\lambda} - m) h(m,\lambda)d\lambda dm}{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\overline{\lambda}} h(m,\lambda)d\lambda dm}
\end{aligned}$$

FIGURE A1

PROOF OF PROPOSITION 4 WHEN $p \in [\mathbb{E}[M] + k\underline{\lambda}, \mathbb{E}[M] + k\overline{\lambda}]$ AND $k < k_1 \equiv \min\{\kappa, \hat{k}\}$ [Color figure can be viewed at wileyonlinelibrary.com]



$$
\begin{aligned}
&= \frac{\int_{\underline{m}}^{p-k\underline{\lambda}} m f(m)dm}{\int_{\underline{m}}^{p-k\underline{\lambda}} f(m)dm} + \frac{\int_{p-k\overline{\lambda}}^{p-k\underline{\lambda}} \int_{\frac{p-m}{k}}^{\overline{\lambda}} (p - k\underline{\lambda} - m)h(m,\lambda)d\lambda dm}{\int_{\underline{m}}^{p-k\underline{\lambda}} \int_{\underline{\lambda}}^{\overline{\lambda}} h(m,\lambda)d\lambda dm} \\
&< \mathbb{E}[M|M \le p - k\underline{\lambda}] + kR_\Lambda \\
&\le \mathbb{E}[M|M \le \mathbb{E}[M] + kR_\Lambda] + kR_\Lambda,
\end{aligned}
$$

where the first inequality follows from the fact that we are adding the set of consumers whose risk type m is such that $m \in [p - k\overline{\lambda}, p - k\underline{\lambda}]$ and whose WTP is above the premium, while assuming that their risk types were all $p - k\underline{\lambda}$; and the second inequality follows from the fact that in the integrand of the numerator in the second term, $p - k\underline{\lambda} - m \le kR_\Lambda$, because $m \ge p - k\overline{\lambda}$.

Now, consider $\eta(k) \equiv \mathbb{E}[M|M \le \mathbb{E}[M] + kR_\Lambda] + kR_\Lambda$. It is clear that $\eta(k)$ is increasing in k; moreover,

$$
\begin{aligned}
\eta(0) &= \mathbb{E}[M|M \le \mathbb{E}[M]] < \mathbb{E}[M]; \\
\eta\left(\frac{\overline{m} - \mathbb{E}[M]}{R_\Lambda}\right) &= \overline{m} > \mathbb{E}[M].
\end{aligned}
$$

Let $k_2 \in (0, \frac{\overline{m}-\mathbb{E}[M]}{R_\Lambda})$ be the unique solution to the equation $\eta(k) = \mathbb{E}[M]$. Then, $\eta(k) < \mathbb{E}[M]$ for all $k < k^\dagger \equiv \min\{k_1, k_2\}$. This completes the proof. ∎

□ **Proof of Proposition 5.**

Proof. We first pin down the monopoly price in the limit. Note that the monopoly firm will choose price p^m to:

$$
\max_{\{p\}} \pi(p; k) \equiv \int_{\theta \in \mathcal{B}(p) \equiv \{(m,\lambda): m + k\lambda \ge p\}} (p - m)dH(m, \lambda). \tag{A1}
$$

As we are interested in how the parameter k affects the emergence of the negative correlation property in equilibrium, we use $p^m(k)$ to denote the monopolist's profit-maximizing premium where k is the parameter measuring the importance of risk preference as a determinant of insurance demand. For what follows, it is useful to define

$$
\lambda^* \equiv \arg\max \lambda[1 - G(\lambda)], \tag{A2}
$$

where $G(\cdot)$ is the marginal CDF of Λ. We assume that λ^* is unique and $\lambda^* \in (\underline{\lambda}, \overline{\lambda})$. To proceed, it is useful to prove the following intermediate result.

Lemma 3. $\lim_{k\to\infty} \frac{p^m(k)}{k} = \lambda^*$.

Proof. It is equivalent to prove that for any arbitrarily small $\epsilon > 0$, there exists a threshold k_3 such that $\frac{p^m(k)}{k} \in (\lambda^* - \epsilon, \lambda^* + \epsilon)$ for $k > k_3$. Without loss of generality, we assume $\epsilon < \min\{\overline{\lambda} - \lambda^*, \lambda^* - \underline{\lambda}\}$.

First, notice that:

$$\left(\lambda^* - \frac{\overline{m}}{k}\right)\left[1 - G\left(\lambda^* - \frac{m}{k}\right)\right] \leq \frac{1}{k}\pi(k\lambda^*; k) = \int_{\lambda^* \leq \frac{m}{k}+\lambda} \left(\lambda^* - \frac{m}{k}\right) dH(m, \lambda)$$
$$\leq \lambda^*\left[1 - G\left(\lambda^* - \frac{\overline{m}}{k}\right)\right].$$

Taking limit of the above inequality yields:

$$\lim_{k\to\infty} \frac{1}{k}\pi(k\lambda^*; k) = \lambda^*[1 - G(\lambda^*)].$$

Second, when $k > m/(\lambda^* - \underline{\lambda} - \epsilon)$, $\lambda^* - \epsilon > \frac{m}{k} + \underline{\lambda}$ and $\frac{\overline{m}}{k} + \overline{\lambda} > \lambda^* + \epsilon$ hold. For $\frac{p}{k}$ that is outside the neighborhood of λ^*, that is, $\frac{p}{k} \in [\frac{m}{k} + \underline{\lambda}, \lambda^* - \epsilon] \cup [\lambda^* + \epsilon, \frac{\overline{m}}{k} + \overline{\lambda}]$, we have

$$\frac{1}{k}\pi(p; k) = \int_{\frac{p}{k} \leq \frac{m}{k}+\lambda} \left(\frac{p}{k} - \frac{m}{k}\right) dH(m, \lambda) \leq \frac{p}{k}\left[1 - G\left(\frac{p}{k} - \frac{\overline{m}}{k}\right)\right].$$

Now, let $\delta \equiv \lambda^*[1 - G(\lambda^*)] - \max_{[\frac{m}{k}+\underline{\lambda}, \lambda^*-\epsilon]\cup[\lambda^*+\epsilon, \frac{\overline{m}}{k}+\overline{\lambda}]} \lambda[1 - G(\lambda)] > 0$. By continuity, there exists a threshold $\check{k}$ such that $\frac{p}{k}[1 - G(\frac{p}{k} - \frac{\overline{m}}{k})] \leq \lambda^*[1 - G(\lambda^*)] - \frac{\delta}{2}$ for $k > \check{k}$. Therefore, for $k > k_3 \equiv \max\{m/(\lambda^* - \underline{\lambda} - \epsilon), \check{k}\}$, the profit at price $p = k\lambda^*$ is greater than the profit at any price p such that $\frac{p}{k} \in [\frac{m}{k} + \underline{\lambda}, \lambda^* - \epsilon] \cup [\lambda^* + \epsilon, \frac{\overline{m}}{k} + \overline{\lambda}]$. This completes the proof. ■

It is useful to point out that Lemma 3 does not depend on the assumption that M and Λ exhibit negative quadrant dependence. Now, we can prove Proposition 5. The average risk of the uninsured is given by:

$$\mathbb{E}[M|\mathcal{N}B(p^m(k))] = \frac{\int_{\frac{p^m(k)}{k} \geq \frac{m}{k}+\lambda} m dH(m, \lambda)}{\int_{\frac{p^m(k)}{k} \geq \frac{m}{k}+\lambda} dH(m, \lambda)}$$
$$> \frac{\int_{\underline{m}}^{\overline{m}} \int_{\underline{\lambda}}^{\frac{p^m(k)-m}{k}} m h(m, \lambda) d\lambda dm + \int_{\underline{m}}^{\overline{m}} \int_{\frac{p^m(k)-m}{k}}^{\frac{p^m(k)-\underline{m}}{k}} \underline{m} h(m, \lambda) d\lambda dm}{\int_{\underline{m}}^{\overline{m}} \int_{\underline{\lambda}}^{\frac{p^m(k)-m}{k}} h(m, \lambda) d\lambda dm + \int_{\underline{m}}^{\overline{m}} \int_{\frac{p^m(k)-m}{k}}^{\frac{p^m(k)-\underline{m}}{k}} h(m, \lambda) d\lambda dm}$$
$$> \frac{\int_{\underline{\lambda}}^{\frac{p^m(k)-m}{k}} m g(\lambda) d\lambda}{G\left(\frac{p^m(k)-\underline{m}}{k}\right)} - (\overline{m} - \underline{m})\left[1 - \frac{G\left(\frac{p^m(k)-\overline{m}}{k}\right)}{G\left(\frac{p^m(k)-\underline{m}}{k}\right)}\right]$$
$$= \mathbb{E}\left[M|\lambda \leq \frac{p^m(k) - m}{k}\right] - (\overline{m} - \underline{m})\left[1 - \frac{G\left(\frac{p^m(k)-\overline{m}}{k}\right)}{G\left(\frac{p^m(k)-\underline{m}}{k}\right)}\right],$$

where the first inequality follows from the fact that we are adding the set of consumers whose risk-aversion type λ is such that $\lambda \in [(p^m(k) - \overline{m})/k, (p^m(k) - \underline{m})/k]$, and whose WTP is above the premium, while assuming that their risk types were all $\underline{m}$; and the second inequality follows from the fact that $m - \underline{m} < \overline{m} - \underline{m}$ and $\int_{\underline{m}}^{\overline{m}} \int_{\frac{p^m(k)-m}{k}}^{\frac{p^m(k)-\underline{m}}{k}} h(m, \lambda) d\lambda dm < G(\frac{p^m(k)-\underline{m}}{k}) - G(\frac{p^m(k)-\overline{m}}{k})$.

By Lemma 3, $\lim_{k\to\infty} \frac{p^m(k)-\underline{m}}{k} = \lim_{k\to\infty} \frac{p^m(k)-\overline{m}}{k} = \lambda^*$. Hence,

$$\lim_{k\to\infty} \left\{\mathbb{E}\left[M|\Lambda \leq \frac{p^m(k) - \underline{m}}{k}\right] - (\overline{m} - \underline{m})\left[1 - \frac{G\left(\frac{p^m(k)-\overline{m}}{k}\right)}{G\left(\frac{p^m(k)-\underline{m}}{k}\right)}\right]\right\} = \mathbb{E}[M|\Lambda \leq \lambda^*].$$

From Definition 3, we must have

$$\Pr(M \leq m|\Lambda \leq \lambda) < \Pr(M \leq m) \text{ for all } (m, \lambda),$$

which implies that $\mathbb{E}[M|\Lambda \leq \lambda^*] > \mathbb{E}[M]$. Hence, there exists a threshold $k^{\dagger\dagger}$ such that $\mathbb{E}[M|\mathcal{N}B(p^m(k))] > \mathbb{E}[M]$ for $k > k^{\dagger\dagger}$. ■

□ **Proof of Proposition 6.**

Proof.

(i) Suppose that $k < 1$. First, consider $\Pi(m, \lambda)$, which is the joint distribution when M and Λ are independent, which trivially satisfies the definition of positive stochastic monotonicity dependence (see Definition 2). Thus, Proposition 3 applies; and thus, for all $p \in (0, k+1)$, we must have[45]:

$$\frac{\int_{\theta\in\mathcal{B}(p)} m d\Pi(m,\lambda)}{\int_{\theta\in\mathcal{B}(p)} d\Pi(m,\lambda)} > \mathbb{E}[M]$$
$$\Leftrightarrow \int_{\theta\in\mathcal{B}(p)} m d\Pi(m,\lambda) > \mathbb{E}[M]\int_{\theta\in\mathcal{B}(p)} d\Pi(m,\lambda). \tag{A3}$$

Now, consider $\mathcal{W}(m, \lambda)$. For $p \in (0, k]$, we have

$$\frac{\int_{\theta\in\mathcal{B}(p)} m d\mathcal{W}(m,\lambda)}{\int_{\theta\in\mathcal{B}(p)} d\mathcal{W}(m,\lambda)} = \frac{\int_{\max\left\{\frac{p-k}{1-k},0\right\}}^{1} m dm}{\int_{\max\left\{\frac{p-k}{1-k},0\right\}}^{1} dm} = \frac{1}{2}\left[\max\left\{\frac{p-k}{1-k},0\right\}+1\right] \geq \frac{1}{2} = \mathbb{E}[M].$$

For $p \in [k, k+1)$, we have $\int_{\theta\in\mathcal{B}(p)} m d\mathcal{W}(m,\lambda) = \int_{\theta\in\mathcal{B}(p)} d\mathcal{W}(m,\lambda) = 0$. Combining these, we conclude that for all $p \in (0, k+1)$:

$$\int_{\theta\in\mathcal{B}(p)} m d\mathcal{W}(m,\lambda) \geq \mathbb{E}[M]\int_{\theta\in\mathcal{B}(p)} d\mathcal{W}(m,\lambda). \tag{A4}$$

Note that $H(m, \lambda; \mu) = \mu\mathcal{W}(m, \lambda) + (1-\mu)\Pi(m, \lambda)$, the average risk of the insured can be bounded from below by

$$\begin{aligned}
\mathbb{E}[M|\mathcal{B}(p)] &= \frac{\int_{\theta\in\mathcal{B}(p)} m dH(m,\lambda)}{\int_{\theta\in\mathcal{B}(p)} dH(m,\lambda)} \\
&= \frac{(1-\mu)\int_{\theta\in\mathcal{B}(p)} m d\Pi(m,\lambda) + \mu\int_{\theta\in\mathcal{B}(p)} m d\mathcal{W}(m,\lambda)}{(1-\mu)\int_{\theta\in\mathcal{B}(p)} d\Pi(m,\lambda) + \mu\int_{\theta\in\mathcal{B}(p)} d\mathcal{W}(m,\lambda)} \\
&> \frac{(1-\mu)\mathbb{E}[M]\int_{\theta\in\mathcal{B}(p)} d\Pi(m,\lambda) + \mu\mathbb{E}[M]\int_{\theta\in\mathcal{B}(p)} d\mathcal{W}(m,\lambda)}{(1-\mu)\int_{\theta\in\mathcal{B}(p)} d\Pi(m,\lambda) + \mu\int_{\theta\in\mathcal{B}(p)} d\mathcal{W}(m,\lambda)} = \mathbb{E}[M],
\end{aligned}$$

where the second equality follows from the definition of $H(\cdot,\cdot)$, and the inequality follows from (A3) and (A4).

(ii) Suppose $k > 1$. The profit function can be derived as[46]:

$$\pi(p;\mu) = \begin{cases} \left(p-\frac{1}{2}\right) - (1-\mu)\frac{1}{3k}p^3 & \text{for} \quad p \in [0,1] \\ (1-\mu)\left[\left(p-\frac{1}{2}\right) + \frac{-3p^2+3p-1}{3k}\right] + \frac{1}{2}\mu\frac{k-p}{k-1}\left(2p - \frac{k-p}{k-1}\right) & \text{for} \quad p \in [1,k] \\ (1-\mu)\left[\frac{(p-1)^3}{3k} - \frac{(p-1)^2}{2} + \frac{k^2}{6}\right] & \text{for} \quad p \in [k, k+1]. \end{cases} \tag{A5}$$

Notice that $\partial\pi/\partial p = 1-(1-\mu)p^2/k > 0$ for $p \in [0,1]$ and $\partial\pi/\partial p = (1-\mu)(p-1)[(p-1)/k-1] < 0$ for $p \in [k, k+1]$. Moreover, $\partial^2\pi/\partial p^2 < 0$ for $p \in [1,k]$. Therefore, $p^m(\mu)$ is the solution to $\partial\pi/\partial p = 0$ for $p \in [1,k]$. Solving for p^m yields,

$$p^m(\mu) = \frac{\mu\frac{k^2}{k-1} + (1-\mu)\left(k-\frac{1}{k}\right)}{\mu\left(2+\frac{1}{k-1}\right) + 2(1-\mu)\left(1-\frac{1}{k}\right)}.$$

The market size is,

$$D(p^m(\mu)) = \mu\left(\frac{k-p^m(\mu)}{k-1}\right) + (1-\mu)\left\{1+\frac{1}{k}\left[\frac{1}{2} - p^m(\mu)\right]\right\}.$$

It can be verified that $dp^m(\mu)/d\mu < 0$ and $dD(p^m(\mu))/d\mu > 0$. Finally, the average risk of the insured can be derived as

$$\mathbb{E}[M|\mathcal{B}(p^m(\mu))] = \frac{\frac{1}{2}\mu\left[\frac{k-p^m(\mu)}{k-1}\right]^2 + (1-\mu)\left\{\frac{1}{2}+\frac{1}{k}\left[\frac{1}{3}-\frac{1}{2}p^m(\mu)\right]\right\}}{\mu\left[\frac{k-p^m(\mu)}{k-1}\right] + (1-\mu)\left\{1+\frac{1}{k}\left[\frac{1}{2}-p^m(\mu)\right]\right\}}.$$

---

[45] Note under the assumed support of M and Λ, the maximum WTP for insurance $\overline{m} + k\overline{\lambda}$ is $k+1$.

[46] See the online web Appendix D for details of the derivations of the profit function, the demand curve, and the cost curves.

Carrying out the algebra, it can be verified that $\mathbb{E}[M|\mathcal{B}(p^m(\mu))] < \mathbb{E}[M]$ is equivalent to

$$[k - p^m(\mu)][p^m(\mu) - 1] - \frac{1}{6}\frac{1-\mu}{\mu}\frac{(k-1)^2}{k} > 0.$$

Denote the left-hand side of the inequality as $\chi(\mu)$. Then, we have

$$\frac{d\chi(\mu)}{d\mu} = 2\left(\frac{k+1}{2} - p^m(\mu)\right)\frac{dp^m(\mu)}{d\mu} + \frac{1}{6\mu^2}\frac{(k-1)^2}{k} > 0,$$

where the inequality follows from the fact that $dp^m(\mu)/d\mu < 0$ and $p^m(\mu) > p^m(1) = k^2/(2k-1) > (k+1)/2$. Therefore, $\chi(\mu)$ is strictly increasing in μ; moreover,

$$\lim_{\mu\to 0}\chi(0) = -\infty < 0;$$

$$\chi(1) = \frac{k(k-1)^3}{2k-1} > 0.$$

Let $\mu^\dagger$ be the unique solution to the equation $\chi(\mu) = 0$. Then, $\mathbb{E}[M|\mathcal{B}(p^m(\mu))] > \mathbb{E}[M]$ for all $\mu < \mu^\dagger$ and $\mathbb{E}[M|\mathcal{B}(p^m(\mu))] < \mathbb{E}[M]$ for all $\mu > \mu^\dagger$. This completes the proof. ■

□ **Proof of Lemma 1.**

Proof. Because $x_i, x_j, x_0 \in \tilde{X}(p^*)$, both $\mathcal{I}_{i0}(m)$ and $\mathcal{I}_{j0}(m)$ cut the support $[\underline{m}, \overline{m}] \times [\underline{\lambda}, \overline{\lambda}]$ into two pieces. Moreover, we have that $p_0^* = 0$ due to the zero-profit condition of the null contract x_0. Suppose to the contrary that $p_i^*/x_i \le p_j^*/x_j$. We consider three cases depending on whether $\mathcal{I}_{i0}(m)$ and $\mathcal{I}_{j0}(m)$ intersect.

Case I $\mathcal{I}_{i0}(m)$ and $\mathcal{I}_{j0}(m)$ intersect at $(\hat{m}, \hat{\lambda}) \in [\underline{m}, \overline{m}] \times [\underline{\lambda}, \overline{\lambda}]$. Note that if $\mathcal{I}_{i0}(m)$ and $\mathcal{I}_{j0}(m)$ intersect at $(\hat{m}, \hat{\lambda})$, then type-$(\hat{m}, \hat{\lambda})$ consumer must be indifferent between contracts (x_i, p_i) and (x_j, p_j), thus, $\mathcal{I}_{ij}(m)$ also intersects both $\mathcal{I}_{i0}(m)$ and $\mathcal{I}_{j0}(m)$ at $(\hat{m}, \hat{\lambda})$. This case is depicted by Figure A2(a). From Assumption 5, the set of consumers selecting x_j can be rewritten as:

$$\begin{aligned}
\mathcal{B}_j(\boldsymbol{p}^*) &\equiv \left\{\theta : U\left(\theta; x_j, p_j^*\right) \ge \max_{j<i}\left\{U\left(\theta; x_i, p_i^*\right)\right\}, U\left(\theta; x_j, p_j^*\right) > \max_{j>i}\left\{U\left(\theta; x_i, p_i^*\right)\right\}\right\} \\
&\subseteq \left\{\theta : U\left(\theta; x_j, p_j^*\right) \ge U\left(\theta; x_0, p_0^*\right), U\left(\theta; x_j, p_j^*\right) \ge U\left(\theta; x_i, p_i^*\right)\right\} \\
&= \left\{(m, \lambda) : \lambda \ge \mathcal{I}_{j0}(m), \lambda \le \mathcal{I}_{ij}(m), \underline{m} \le m \le \overline{m}, \underline{\lambda} \le \lambda \le \overline{\lambda}\right\} \\
&= \left\{(m, \lambda) : \underline{m} \le m \le \hat{m}, \mathcal{I}_{j0}(m) \le \lambda \le \min\left\{\overline{\lambda}, \mathcal{I}_{ij}(m)\right\}\right\} := \mathcal{B}'_j(\boldsymbol{p}^*).
\end{aligned}$$

The shaded region of Figure A2(a) illustrates $\mathcal{B}'_j(\boldsymbol{p}^*)$, which can be decomposed into two groups: those who only prefer contract (x_j, p_j) over not purchasing (the dark dotted region in Figure A2(a)) and those who prefer both (x_i, p_i) and (x_j, p_j) over not purchasing, but prefer (x_j, p_j) over (x_i, p_i) (the light dotted region in Figure A2(a)).

Next, notice that type-$(\hat{m}, \hat{\lambda})$ consumer is indifferent between contracts $(0, 0)$ and (x_i, p_i), we must have that $p_i - 0 \ge (x_i - 0)\hat{m}$ from Assumption 4, which is equivalent to $\hat{m} \le p_i^*/x_i$. Therefore, we have that

$$\mathbb{E}\left[M|\mathcal{B}_j(\boldsymbol{p}^*)\right] < \hat{m} \le \frac{p_i^*}{x_i} \le \frac{p_j^*}{x_j},$$

which is a contradiction to the zero-profit condition required for contract x_j.

Case II $\mathcal{I}_{i0}(m)$ *lies above* $\mathcal{I}_{j0}(m)$. This case is depicted by Figure A2(b). First, note that $\mathcal{I}_{ij}(m)$ must cut the support $[\underline{m}, \overline{m}] \times [\underline{\lambda}, \overline{\lambda}]$ into two pieces. Otherwise, either $D_i(\boldsymbol{p}^*) = 0$ or $D_j(\boldsymbol{p}^*) = 0$, a contradiction. Second, from the definition of indifference curve, $\mathcal{I}_{ij}(m)$ does not intersect with either $\mathcal{I}_{i0}(m)$ or $\mathcal{I}_{j0}(m)$. Together with Assumption 5, $\mathcal{I}_{ij}(m)$ must lie above $\mathcal{I}_{i0}(m)$ (see Figure A2(b)), and the risk type of consumers selecting x_j can be bounded from above by $\check{m}$, where $\check{m} = \mathcal{I}_{ij}^{-1}(\underline{\lambda})$ if $U(\overline{m}, \underline{\lambda}; x_i, p_i^*) > U(\overline{m}, \underline{\lambda}; x_j, p_j^*)$, and $\check{m} = \overline{m}$ otherwise. By definition, type-$(\check{m}, \underline{\lambda})$ consumer weakly prefers contract (x_j, p_j^*) to (x_i, p_i^*). Together with Assumption 4, we must have that

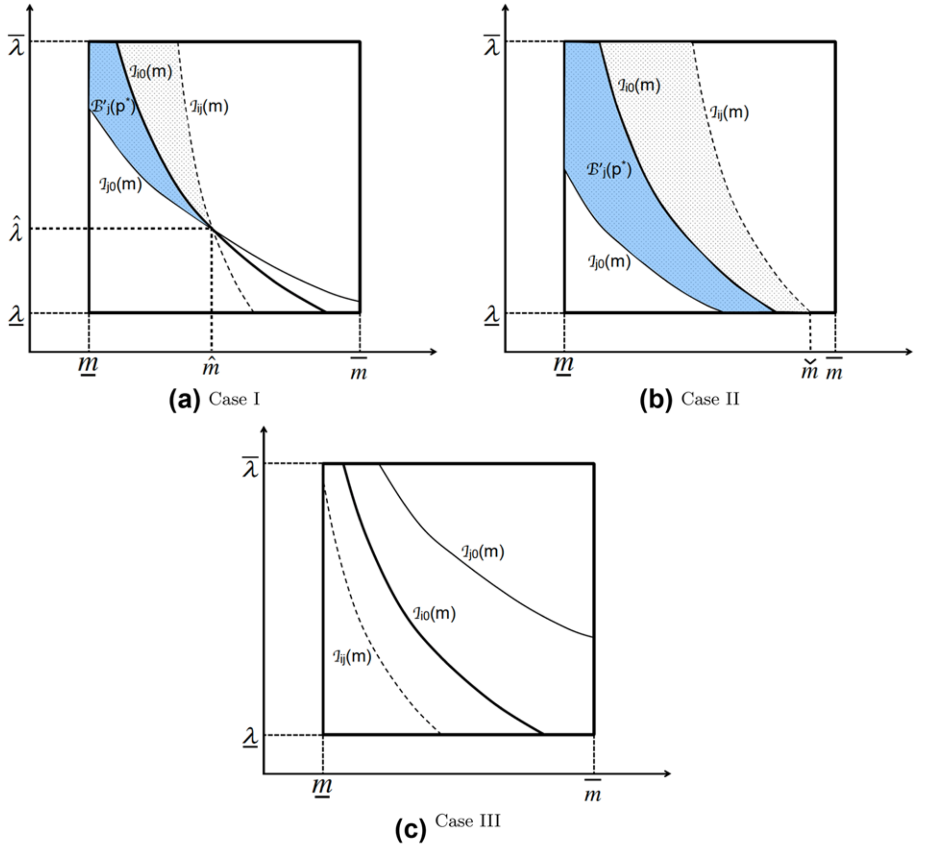
$$p_i^* - p_j^* \ge (x_i - x_j)\check{m}.$$

Therefore, $\check{m}$ can be bounded from above by,

$$\check{m} \le \frac{p_i^* - p_j^*}{x_i - x_j} \le \frac{p_j^*}{x_j},$$

FIGURE A2

PROOF OF LEMMA 1 [Color figure can be viewed at wileyonlinelibrary.com]



(a) Case I (b) Case II (c) Case III

where the second inequality follows from the postulated $p_i^*/x_i \leq p_j^*/x_j$. Therefore,

$$\mathbb{E}\left[M|\mathcal{B}_j(\boldsymbol{p}^*)\right] < \check{m} \leq \frac{p_j^*}{x_j},$$

which is a contradiction to the zero-profit condition required for contract (x_j, p_j^*).

Case III $\mathcal{I}_{i0}(m)$ *lies below* $\mathcal{I}_{j0}(m)$. This case is depicted by Figure A2(c). By the same argument as in Case II, $\mathcal{I}_{ij}(m)$ does not intersect with either $\mathcal{I}_{i0}(m)$ or $\mathcal{I}_{j0}(m)$, and $\mathcal{I}_{ij}(m)$ must lie below $\mathcal{I}_i(m)$ (see Figure A2(c)). This implies that a consumer whose type is above or on $\lambda = \mathcal{I}_{i0}(m)$ will purchase contract (x_i, p_i^*), and will end up with no insurance otherwise. Therefore, there will be no insurance demand for contract(x_j, p_j^*), a contradiction to $D_j(\boldsymbol{p}^*) > 0$.

■

□ **Proof of Proposition 7.**

Proof. Suppose $|\tilde{X}(\boldsymbol{p}^*)| = 2$. Then, Proposition 1 applies and the positive correlation property must hold. Suppose $|\tilde{X}(\boldsymbol{p}^*)| \geq 3$, for $x_i > x_j > 0$, we must have,

$$\mathbb{E}\left[M|(m,\lambda) \in \mathcal{B}_i(\boldsymbol{p}^*)\right] = \frac{p_i^*}{x_i} > \frac{p_j^*}{x_j} = \mathbb{E}\left[M|(m,\lambda) \in \mathcal{B}_j(\boldsymbol{p}^*)\right],$$

where the two equalities follow from the zero-profit condition required for contract x_i and x_j, and the strict inequality follows from Lemma 1.

Let $x_s \equiv \min\{\tilde{X}(\boldsymbol{p}^*)\backslash\{x_0\}\}$. It remains to be shown that

$$\mathbb{E}\left[M|(m,\lambda) \in \mathcal{B}_s(\boldsymbol{p}^*)\right] > \mathbb{E}\left[M|(m,\lambda) \in \mathcal{B}_0(\boldsymbol{p}^*)\right].$$

To prove this, suppose there is only one contract of quality x_s available on the market where consumer characteristics are drawn from the set $\mathcal{B}_s(\boldsymbol{p}^*) \cup \mathcal{B}_0(\boldsymbol{p}^*)$. It is obvious that p_s^* is an equilibrium price in such a market. Moreover, the set of buyers and the set of nonbuyers of contract x_s are $\mathcal{B}_s(\boldsymbol{p}^*)$ and $\mathcal{B}_0(\boldsymbol{p}^*)$, respectively. We can thus apply Proposition 1 to conclude $\mathbb{E}[M|(m,\lambda) \in \mathcal{B}_s(\boldsymbol{p}^*)] > \mathbb{E}[M|(m,\lambda) \in \mathcal{B}_0(\boldsymbol{p}^*)]$. This completes the proof. ■

□ **Proof of Proposition 8.**

Proof. From Lemma 2, $\pi(p, x; k) = x\pi(p/x, 1; (2-x)k)$. Notice that $(2-x)k < 2k$. Applying Proposition 4 with $\hat{k}^\dagger = \frac{1}{2}k^\dagger$ completes the proof. ■

□ **Proof of Proposition 9.**

Proof. From Lemma 2, $\pi(p, x; k) = x\pi(p/x, 1; (2-x)k)$. Notice that $(2-x)k \geq k$. Applying Proposition 5 with $\hat{k}^{\dagger\dagger} = k^{\dagger\dagger}$ completes the proof. ■

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Supporting information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

web Appendix