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Incentives in lottery contests with draws☆

Shanglyu Deng [a], Xiruo Wang [b,*], Zenan Wu [a,*]

[a] School of Economics, Peking University, Beijing 100871, China
[b] School of Economics and Management, Tsinghua University, Beijing 100084, China

HIGHLIGHTS

- A private value lottery contest model with draws is considered.
- Draws increase (decrease) the strong (weak) contestant's effort incentive.
- Total effort is reduced after the introduction of a draw.
- Expected winner's effort can be higher if types are sufficiently dispersed.

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ABSTRACT

We investigate the incentive consequences of introducing the possibility of draws into lottery contests. Equilibrium total effort unambiguously decreases when draws are introduced, whereas the equilibrium expected winner's effort increases when the contestants' valuations of the prize become sufficiently dispersed.

1. Introduction

Contests that permit draws (or interchangeably, ties, gaps), in which no contestant wins the prize outright, are commonly observed in practice and have been extensively studied in the contest literature.[1] Intuitively, adding the possibility of a draw in a contest or a tournament softens competition between contestants and hence dampens their incentive to exert effort. This intuition is formalized and confirmed by Nti (1997) using a symmetric multi-player Tullock contest.

Even though draws reduce player incentive, previous studies have pointed out the potential merits of introducing draws into a contest/tournament. For instance, Nalebuff and Stiglitz (1983) and Eden (2007) show that although the introduction of draws weakens incentive, it can be optimal to the designer because each actual draw occurrence saves a payment to a contestant and increases the designer's expected profits. Recently, Imhof and Kräkel (2014) show that the designer strictly benefits from a gap if contestants are risk-averse because it provides partial insurance to the contestants on their income distributions and helps reduce the agency cost.

To the best of our knowledge, the extant literature has assumed that players are identical and has restricted its attention to the symmetric equilibria. In this paper, we relax the symmetry assumption and investigate the impact of the contestants' heterogeneity on their effort incentives. Specifically, we build a model based on Nti (1997) by allowing for heterogeneity among contestant winning values, in which no contract is provided and all agents

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* Corresponding authors.
E-mail addresses: dengshanglv@pku.edu.cn (S. Deng), wangxr3.14@sem.tsinghua.edu.cn (X. Wang), zenan@pku.edu.cn (Z. Wu).

[1] For a list that is indicative, but by no means exhaustive, see Nalebuff and Stiglitz (1983), Lazear and Rosen (1981), Eden (2007), Imhof and Kräkel (2014, 2016) for gaps in rank-order tournaments; see Nti (1997), Blavatskyy (2010), Jia (2012), Vesperoni and Yildizparlak (2017) for draws in imperfectly discriminating contests; see Gelder et al. (2015, 2016) for ties in perfectly discriminating contests (or equivalently, all-pay auctions).

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are risk-neutral. This setup allows us to abstract from the aforementioned wage-saving and insurance-provision motives from the designer, and to focus on the impact of draws on incentives.

We find that draws can benefit the designer from a pure incentive perspective. When there are two heterogeneous contestants, introducing a draw increases the strong player's effort incentives and decreases the weak player's effort level (Lemma 1). On the whole, total effort decreases with the presence of draws; however, if the players' types are sufficiently dispersed, the equilibrium expected winner's effort increases (Proposition 1). This result can be generalized to contests with more than two players (Proposition 2).

2. Model

There are $n \geq 2$ risk-neutral contestants competing for a prize. The value of the prize to contestant $i \in \{1, \dots, n\}$ is $v_i > 0$, which is common knowledge. Without loss of generality, we order the contestants according to their valuations of the prize so that $v_1 \geq v_2 \geq \cdots \geq v_n > 0$. To win the prize, contestants exert irreversible effort simultaneously. Following Nti (1997), we assume that the winning probability of contestant i under effort profile $(x_1, \dots, x_n)$ is given by

$$p_i(x_1, \dots, x_n) = \begin{cases} \dfrac{x_i}{\sum_{i=1}^n x_i + s} & \text{if } \sum_{i=1}^n x_i + s > 0 \\ \dfrac{1}{n} & \text{otherwise,} \end{cases}$$

where $s \geq 0$. Blavatskyy (2010) recently axiomatized this functional form.[2] The parameter s allows us to accommodate the possibility of a draw in a simple manner.[3] When $s = 0$, the winning probabilities across all contestants add up to one (i.e., $\sum_{i=1}^n p_i = 1$) and hence the prize is allocated to one of the contestants with certainty. When $s > 0$, a draw occurs with positive probability (i.e., $\frac{s}{\sum_{i=1}^n x_i + s} > 0$) and no contestant wins the prize.[4]

We assume that the contest designer's objective is to either (i) maximize the total effort of all contestants or (ii) maximize the expected winner's effort. The first objective function is commonly assumed in the contest literature. The second objective function is relevant in many contexts (e.g., research competitions) where the contest designer may care most about the performance of the winning contestant because only the winner's project will be executed (Baye and Hoppe, 2003; Serena, 2017).[5]

It is useful to introduce some notations before we proceed. Fixing s, denote contestant i's equilibrium effort by $x_i^*(s)$ for $i \in \{1, \dots, n\}$. Similarly, denote the equilibrium total effort and expected winner's effort by $TE(s)$ and $WE(s)$ respectively. By definition, we have that

$$TE(s) \equiv \sum_{i=1}^n x_i^*(s), \tag{1}$$

and

$$WE(s) \equiv \sum_{i=1}^n p_i(x_1^*, \dots, x_n^*) \cdot x_i^*(s) = \frac{\sum_{i=1}^n \left[x_i^*(s)\right]^2}{\sum_{i=1}^n x_i^*(s) + s}. \tag{2}$$

2 See also Jia (2012); Jia et al. (2013) for a stochastic derivation.

3 The parameter s is treated as a discount rate in a patent race context in Nti (1997).

4 Alternatively, we can assume that each contestant receives an identical fraction of the prize in the case of a draw. Please see Footnote 6 for more discussions.

5 The objective of maximizing the expected winner's effort in the imperfectly discriminating contest is parallel to the widely adopted objective of maximizing the expected highest effort in the perfectly discriminating contest (i.e., an all-pay auction).

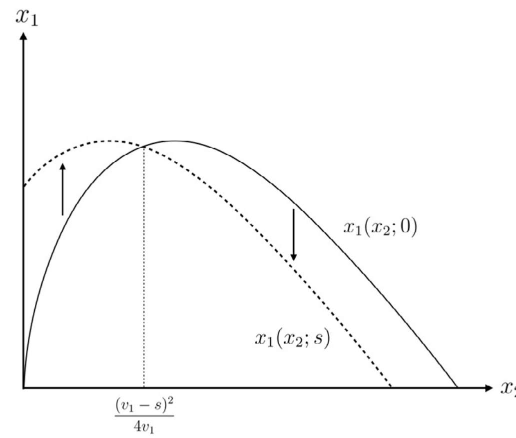


Fig. 1. Contestant 1's best response function.

3. Contest with $n = 2$ players

To explain the intuitions most cleanly, let us first consider a contest with two contestants. Fixing $s \geq 0$ and contestant 2's effort level x_2, contestant 1's response function, denoted by $x_1(x_2; s)$, can be derived as

$$x_1(x_2; s) = \max \left\{ \underbrace{\sqrt{v_1 \cdot (x_2 + s)}}_{\text{indirect effort boost effect}} - \underbrace{(x_2 + s)}_{\text{direct effort loss effect}}, 0 \right\}. \tag{3}$$

Contestant 2's best response function $x_2(x_1; s)$ can be derived similarly. From Eq. (3), introducing the possibility of a draw has two opposing effects. First, as the previous literature has pointed out, it *directly* weakens a contestant's effort incentive due to the decreased probability of winning for any effort level. Second, because the probability of a draw (i.e., $\frac{s}{x_1 + x_2 + s}$) depends on a contestant's effort, it also *indirectly* provides an incentive to a contestant to increase his effort level in order to avoid the loss suffered from a draw. Moreover, the latter indirect effect vanishes and will be dominated by the former direct effect as the rival's effort increases. Simple algebra shows that these two opposing effects cancel out at $x_2 = \frac{(v_1 - s)^2}{4v_1}$. Therefore, contestant 1's effort will increase upon the introduction of a draw when the rival's effort level is low (i.e., $x_2 < \frac{(v_1 - s)^2}{4v_1}$) and will decrease otherwise, as shown in Fig. 1.

The next lemma characterizes the equilibrium effort portfolio when s is sufficiently small.

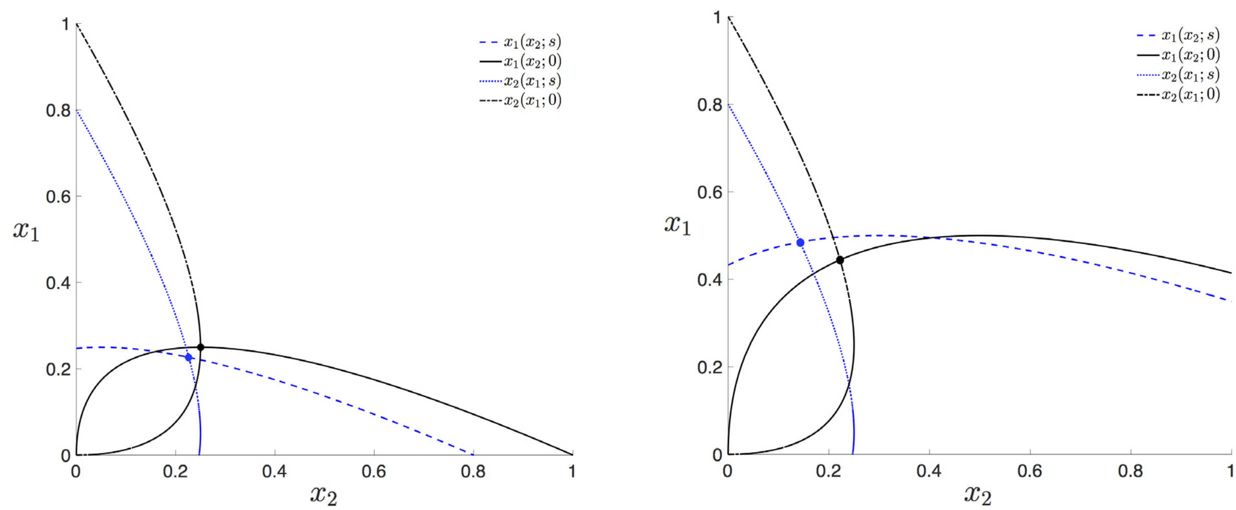
Lemma 1. *Suppose $n = 2$ and $s < \frac{v_2^2}{v_1}$. Then the equilibrium portfolio $(x_1^*(s), x_2^*(s))$ is given by*

$$x_i^*(s) = y^*(s) - \frac{[y^*(s)]^2}{v_i} > 0, \text{ for } i = 1, 2, \tag{4}$$

where

$$y^*(s) = \frac{1 + \sqrt{1 + 4s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}}{2\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}. \tag{5}$$

Moreover, (i) if $v_1 = v_2$, then $x_i^*(s)$ is strictly decreasing in $s \in [0, v_2]$ for $i = 1, 2$; (ii) if $v_1 > v_2$, then $x_1^*(s)$ is strictly increasing in



(a) Symmetric contest: $(v_1, v_2, s) = (1, 1, 0.2)$.

(b) Asymmetric contest: $(v_1, v_2, s) = (2, 1, 0.2)$.

Fig. 2. Equilibrium effort portfolio.

$s \in [0, \bar{s}]$, *and* $x_2^*(s)$ *is strictly decreasing in* $s \in [0, \bar{s}]$, *where* $\bar{s} := \min\left\{ \frac{\left(\frac{v_1}{v_2}\right)^2 - 1}{4\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}, \frac{v_2^2}{v_1} \right\}$.

Proof. Define $y^*(s) := x_1^*(s) + x_2^*(s) + s$ and suppose that $x_1^*(s) > 0$ and $x_2^*(s) > 0$. It follows from (3) that $\left(x_1^*(s), x_2^*(s)\right)$ is the solution to the following system of equations:

$$\begin{cases} x_1^*(s) + x_2^*(s) + s = \sqrt{v_1\left[x_2^*(s) + s\right]} \\ \Leftrightarrow y^*(s) = \sqrt{v_1\left[y^*(s) - x_1^*(s)\right]} \Leftrightarrow x_1^*(s) = y^*(s) - \dfrac{[y^*(s)]^2}{v_1}, \\ x_1^*(s) + x_2^*(s) + s = \sqrt{v_2\left[x_1^*(s) + s\right]} \\ \Leftrightarrow y^*(s) = \sqrt{v_2\left[y^*(s) - x_2^*(s)\right]} \Leftrightarrow x_2^*(s) = y^*(s) - \dfrac{[y^*(s)]^2}{v_2}. \end{cases}$$

Summing up the above equations and solving for $y^*(s)$ yields the expression in (5). For $x_1^*(s)$ and $x_2^*(s)$ to be strictly positive, we need $y^*(s) < \min\{v_1, v_2\} = v_2$ from (4), which is equivalent to $s < \frac{v_2^2}{v_1}$.

Next, it follows from Eq. (4) that

$$\frac{dx_i^*(s)}{ds} = \frac{2}{v_i}\left[\frac{v_i}{2} - y^*(s)\right]\frac{dy^*(s)}{ds}, \quad \text{for } i = 1, 2. \tag{6}$$

Case I: $v_1 = v_2$. It is clear that $\frac{dy^*(s)}{ds} > 0$ from (5). Therefore, we have that

$$\frac{v_1}{2} = \frac{v_2}{2} = y^*(0) < y^*(s), \text{ for } s > 0,$$

where the equalities follow from (5) and the postulated $v_1 = v_2$. Together with (6), we have that $\frac{dx_i^*(s)}{ds} < 0$ for $0 < s < v_2$.

Case II: $v_1 > v_2$. It follows from $\frac{\left(\frac{v_1}{v_2}\right)^2 - 1}{4\sum_{i=1}^2 \frac{1}{v_i}} > 0$ and $\frac{dy^*(s)}{ds} > 0$ that

$$\frac{v_2}{2} < \frac{1}{\sum_{i=1}^2 \frac{1}{v_i}} = y^*(0) < y^*(s) < y^*\left(\frac{\left(\frac{v_1}{v_2}\right)^2 - 1}{4\sum_{i=1}^2 \frac{1}{v_i}}\right) = \frac{v_1}{2},$$
$$\text{for } 0 < s < \bar{s},$$

which in turn implies that $\frac{dx_1^*(s)}{ds} > 0$ and $\frac{dx_2^*(s)}{ds} < 0$ for $0 \le s < \bar{s}$. This completes the proof. ■

Lemma 1 predicts that $x_1(s)$ and $x_2(s)$ move in the same direction only when the two contestants are symmetric as depicted in Fig. 2(a). However, if there exists some heterogeneity between the two contestants, then the strong contestant exerts more effort with the presence of the possibility of a draw than without, while the weaker contestant reduces his effort. The intuition is as follows. Note that the strong contestant exerts more effort than the weak one in equilibrium when $s = 0$. Specifically, $x_1^*(0) = \frac{v_1^2 v_2}{(v_1+v_2)^2} > x_2^*(0) = \frac{v_2^2 v_1}{(v_1+v_2)^2}$. Moreover, recall that contestant 1 exerts more effort after the introduction of a draw when $x_2 < \frac{(v_1-s)^2}{4v_1}$. Fixing contestant 2's effort level at $x_2^*(0)$, because $x_2^*(0) = \frac{v_2^2 v_1}{(v_1+v_2)^2} < \frac{(v_1-s)^2}{4v_1}$ holds when $s > 0$ is sufficiently small, the indirect effort boost effect overrides the direct effort loss effect for the strong contestant, implying his tendency to increase his effort provision. By the same argument, contestant 2 tends to reduce his effort, holding contestant 1's effort fixed at $x_1^*(0)$. Therefore, $x_1^*(s) > x_1^*(0)$ and $x_2^*(s) < x_2^*(0)$ as Fig. 2(b) illustrates.

Lemma 1 states that both contestants' effort levels move in opposite directions in an asymmetric contest when the possibility of a draw is introduced.[6] Therefore, it is unclear how the aggregate effort and the expected winner's effort change accordingly. Our next proposition summarizes the comparative statics of $WE(s)$ and $TE(s)$ in s.

Proposition 1. *Suppose* $n = 2$. *Then* $TE(s) < TE(0)$ *for all* $s \in \left(0, \frac{v_2^2}{v_1}\right)$.[7] *Moreover,* $WE(s) \gtrless WE(0)$ *if* $\frac{v_2}{v_1} \gtrless 2 + \sqrt{3}$ *for all sufficiently small* $s > 0$.

Proof. The proof closely follows that of Proposition 2 below and is omitted for brevity. ■

6 If each contestant receives a positive payment instead of zero in the case of a draw, then the indirect effort boost effect is weakened and as a result both contestants' effort incentives may be reduced upon the introduction of a draw. To see this more clearly, suppose that contestant $i \in \{1, 2\}$ receives $\frac{v_i}{2}$ when a draw occurs. Then contestant 1's best response function in (3) becomes $x_1(x_2; s) = \max\{\sqrt{v_1 \cdot (x_2 + \frac{s}{2})} - (x_2 + s), 0\}$, from which we can see that the corresponding indirect effort boost effect $\sqrt{v_1 \cdot (x_2 + \frac{s}{2})}$ is less than $\sqrt{v_1 \cdot (x_2 + s)}$. Moreover, it can be verified that both $x_1^*(s)$ and $x_2^*(s)$ are strictly decreasing in s for $s < \frac{2v_1 v_2^2}{(v_1+v_2)^2}$.

7 For $s \ge \frac{v_2^2}{v_1}$, contestant 2 does not exert effort in equilibrium after the introduction of a draw.

An effort-maximizing contest designer has no incentive to deviate from $s = 0$ locally. In contrast, the expected winner's effort increases after the introduction of the possibility of draws if contestants' values of winning are sufficiently dispersed. Proposition 1 implies that the weak contestant's decrease in effort always outweighs the strong contestant's increase in effort. Therefore, introducing the possibility of a draw has an unambiguous effect on aggregate effort.

To understand why the possibility of a draw can benefit the designer in terms of the expected winner's effort, recall that the expected winner's effort is equal to $\frac{x_1^2+x_2^2}{x_1+x_2+s}$. Therefore, fixing the amount of total effort, the expected winner's effort increases as the effort profile becomes more and more uneven. This diversified effort profile can be achieved by introducing a small $s > 0$ when contestants are heterogeneous from Lemma 1 and is aligned with the interest of the contest designer.

4. Contest with $n > 2$ players

In this section, we show that the main results of the previous section can be generalized to an environment with more than two contestants. It is useful to point out that both contestants exert a positive amount of effort in equilibrium when $s = 0$ for $n = 2$. However, when $n \geq 3$, it is possible that some contestants may stay inactive (or equivalently, exert zero effort). Define $n^\dagger$ as

$$n^\dagger := \min\left\{\left\{m : v_{m+1} \leq \frac{m-1}{\sum_{i=1}^{m}\frac{1}{v_i}}, m = 2, \ldots, n-1\right\} \cup \left\{n\right\}\right\}. \quad (7)$$

The set of active and inactive players for $s = 0$ is characterized by the following lemma.

Lemma 2 (Fang, 2002). *$x_i^*(0) > 0$ for $i \leq n^\dagger$ and $x_i^*(0) = 0$ for $i > n^\dagger$.*

The next proposition reports a result that is parallel to that of Proposition 1.

Proposition 2. *Suppose $n \geq 2$. Then $TE(s) < TE(0)$ for all sufficiently small $s > 0$. Moreover, $WE(s) > WE(0)$ for all sufficiently small $s > 0$ if*

$$\frac{\sum_{i=1}^{n^\dagger}\frac{1}{v_i^2}}{\left(\sum_{i=1}^{n^\dagger}\frac{1}{v_i}\right)^2} > \frac{3n^\dagger-4}{3(n^\dagger-1)^2}, \quad (8)$$

where $n^\dagger$ is defined in expression (7).

Proof. It is straightforward to verify that $x_i^*(s) = 0$ for $i \geq n^\dagger + 1$ and $x_i^*(s) > 0$ for $i = 1, \ldots, n^\dagger$ when $s > 0$ is sufficiently small. Similar to the proof in Lemma 1, define $y^*(s) := \sum_{i=1}^{n^\dagger} x_i^*(s) + s$. The first-order condition with respect to x_i^* can be written as

$$\frac{1}{v_i} y^*(s)^2 = y^*(s) - x_i^*(s), \text{ for } i \in \{1, \ldots, n^\dagger\}.$$

Summing up the above conditions yields that

$$\left(\sum_{i=1}^{n^\dagger}\frac{1}{v_i}\right) y^*(s)^2 = \sum_{i=1}^{n^\dagger}\left[y^*(s) - x_i^*(s)\right] = \left(n^\dagger - 1\right) y^* + s.$$

Solving for $y^*(s)$ yields the following:

$$y^*(s) = \frac{n^\dagger - 1 + \sqrt{(n^\dagger-1)^2 + 4s\sum_{i=1}^{n^\dagger}\frac{1}{v_i}}}{2\sum_{i=1}^{n^\dagger}\frac{1}{v_i}}, \quad (9)$$

and

$$x_i^*(s) = y^*(s) - \frac{[y^*(s)]^2}{v_i}, \text{ for } i \in \{1, \ldots, n^\dagger\}. \quad (10)$$

Therefore, $TE(s)$ in expression (1) can be rewritten as

$$TE(s) = y^*(s) - s,$$

and $WE(s)$ in expression (2) can be rewritten as

$$WE(s) = \frac{\sum_{i=1}^{n^\dagger}[x_i^*(s)]^2}{y^*(s)} = y^*(s) \cdot \sum_{i=1}^{n^\dagger}\left(1 - \frac{y^*(s)}{v_i}\right)^2.$$

Differentiating $TE(s)$ with respect to s yields that

$$\frac{dTE(s)}{ds} = \frac{1}{\sqrt{(n^\dagger-1)^2 + 4s\sum_{i=1}^{n^\dagger}\frac{1}{v_i}}} - 1 \leq \frac{1}{\sqrt{1 + 4s\sum_{i=1}^{n^\dagger}\frac{1}{v_i}}} - 1 < 0,$$

where the first inequality follows from the observation that $n^\dagger \geq 2$.

For the case $n = 2$, it follows from Lemma 1 that $x_1^*(s) > 0$ and $x_2^*(s) > 0$ for all $s \in \left(0, \frac{v_2^2}{v_1}\right)$. Therefore, the above condition holds for all $s \in \left(0, \frac{v_2^2}{v_1}\right)$, which in turn implies that $TE(s) < TE(0)$ holds for all $s \in \left(0, \frac{v_2^2}{v_1}\right)$ in Proposition 1.

Similarly, we have that

$$\frac{dWE(s)}{ds} = \frac{dy^*(s)}{ds} \cdot \sum_{i=1}^{n^\dagger}\left[1 - \frac{4y^*(s)}{v_i} + \frac{3[y^*(s)]^2}{v_i^2}\right].$$

Note that $\frac{dy^*(s)}{ds} > 0$. Therefore, $\left.\frac{dWE(s)}{ds}\right|_{s=0} > 0$ is equivalent to

$$n^\dagger - 4\sum_{i=1}^{n^\dagger}\frac{1}{v_i}y^*(0) + 3\sum_{i=1}^{n^\dagger}\frac{1}{v_i^2}[y^*(0)]^2 > 0.$$

Together with the fact that $y^*(0) = \frac{n^\dagger-1}{\sum_{i=1}^{n^\dagger}\frac{1}{v_i}}$ from (9), the above inequality can be simplified as

$$\frac{\sum_{i=1}^{n^\dagger}\frac{1}{v_i^2}}{\left(\sum_{i=1}^{n^\dagger}\frac{1}{v_i}\right)^2} > \frac{3n^\dagger-4}{3(n^\dagger-1)^2},$$

which coincides with the expression in (8). This completes the proof. ■

Fixing $n^\dagger$, condition (8) is more likely to be satisfied as the contestants become more heterogeneous. This confirms our intuition for the two-player case in Proposition 1. However, the intuition is more subtle for $n \geq 3$ because the set of active contestants (i.e., $n^\dagger$) may also change when we change the distribution of contestant types.

5. Conclusion

In this paper, we depart from the symmetry assumption that is commonly assumed in the contest and tournament literature on draws. We show that introducing draws into a contest can be optimal to the contest designer who aims to the maximize the expected winner's effort when the contestants are sufficiently heterogeneous.

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