Expectations-Based Loss Aversion in Contests

ONLINE APPENDIX

(Not Intended for Publication)

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In this online appendix, we collect the analyses and discussions omitted from the main text that complement our baseline results.[1] Four extensions are considered. Online Appendix A examines the contest game under strong loss aversion. Online Appendix B studies a contest with a concave impact function. Online Appendix C compares the equilibrium under CPNE to that under PPNE. Finally, Online Appendix D considers a setting in which two contestants differ in not only their prize valuations but also the degrees of loss aversion.

A Strong Loss Aversion

In this section, we discuss the case of strong loss aversion, i.e., $k \equiv \eta(\lambda - 1) > 1/3$. We first show that CPNE may fail to exist when k exceeds the cutoff 1/3. Next, we consider a simple contest design problem in which an effort-maximizing contest designer selects a contender to rival an incumbent player; the case sheds light on the implications of loss aversion for contest design.

A.1 Existence and Uniqueness of CPNE

Let us introduce the notation $y_i := f_i(x_i)$ and define the inverse function of $f_i(\cdot)$ by $\phi_i(\cdot) := f_i^{-1}(\cdot)$. The function $\phi_i(\cdot)$ describes the amount of effort required for contestant i to generate an effective bid $y_i := f_i(x_i)$. We further assume the following.

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[1] This note is not self-contained; it is the online appendix of the paper "Expectations-Based Loss Aversion in Contests."

Assumption A1 *$\phi_i(\cdot)$ is a trice-differentiable function, with $\phi_i'(y_i) > 0$, $\phi_i''(y_i) \geq 0$, $\phi'''(y_i) \geq 0$, and $\phi_i(0) = 0$.*

Note that Assumption 1 implies immediately that $\phi_i'(y_i) > 0$, $\phi_i''(y_i) \geq 0$, and $\phi_i(0) = 0$. Compared with Assumption 1, the additional condition required by Assumption A1 is $\phi'''(y_i) \geq 0$, which is also assumed by Dato, Grunewald, and Müller (2018). Note that Assumption A1 is automatically satisfied if the impact function is linear.

Proposition A1 *Suppose that Assumption A1 is satisfied and $k \equiv \eta(\lambda - 1) \in (\frac{1}{3}, \frac{1}{2}]$. Then either (i) there exists a unique pure-strategy CPNE of the contest game or (ii) there exists no pure-strategy CPNE.*

Proof. The proof closely follows that of Proposition 1. Note that the left-hand side of Equation (6) in the proof of Proposition 1 is quadratic and is inverted U-shaped in y_i on $[0, \infty)$ for $\frac{1}{3} < k \leq \frac{1}{2}$, and the right-hand side is weakly convex and weakly increasing in y_i under Assumption A1. Therefore, the unique solution in the interval $(0, s)$ is guaranteed, implying that both $g_i(s)$ and $\rho_i(s) \equiv g_i(s)/s$ are well defined on $(0, \infty)$.

We first show that $\rho_i'(s)$ in Equation (9) is negative, i.e.,

$$\rho_i'(s) = -\frac{\phi_i'(\rho_i s) + \rho_i s \times \phi_i''(\rho_i s)}{(1 - 3k + 4k\rho_i)\, v_i + s^2 \times \phi_i''(\rho_i s)} < 0.$$

Clearly, the numerator in the above expression is strictly positive because $\phi_i' > 0$ and $\phi_i'' \geq 0$. For the denominator, we have

$$\begin{aligned}
(1 - 3k + 4k\rho_i)\, v_i + s^2 \times \phi_i''(\rho_i s) &\geq (1 - 3k + 4k\rho_i)\, v_i + s \times \frac{\phi_i'(\rho_i s) - \phi_i'(0)}{\rho_i} \\
&> (1 - 3k + 4k\rho_i)\, v_i + \frac{1 - \rho_i}{\rho_i}(1 - k + 2k\rho_i)\, v_i - \frac{1}{\rho_i}(1 - k) v_i \\
&= 2k\rho_i v_i \geq 0,
\end{aligned}$$

where the first inequality follows from $\phi_i''' \geq 0$, as stated in Assumption A1, and the second inequality from (8) and $s < \frac{(1-k)v_i}{\phi_i'(0)}$.

To complete the proof, it remains to show that $\chi(s) := \sum_{i=1}^N \rho_i(s) - 1 = 0$ has at most one positive solution for $\frac{1}{3} < k \leq \frac{1}{2}$. It can be verified that $\rho_i(s)$ is discontinuous at $s = (1 - k)v_i/\phi_i'(0)$ for $\frac{1}{3} < k \leq \frac{1}{2}$. Moreover, $\rho_i(s)$ is continuous and strictly decreasing in s for $s < (1 - k)v_i/\phi_i'(0)$, and is constant for $s \geq (1 - k)v_i/\phi_i'(0)$. Therefore, $\chi(s)$ is strictly decreasing in s for $s \in (0, \frac{(1-k)v_1}{\phi_i'(0)}]$, but is discontinuous at $s = (1 - k)v_i/\phi_i'(0)$ with $i \in \mathcal{N}$. This implies immediately that $\chi(s) = 0$ has at most one positive solution. ■

Proposition A1 eliminates the possibility of multiple equilibria: Whenever a CPNE exists, it must be unique. Interestingly, multiple CPNEs are possible in the framework of Dato et al. (2017). In particular, they show that an asymmetric equilibrium may exist when players are sufficiently loss averse, in which one player exerts no effort and the other player exerts positive effort. Such an equilibrium cannot arise in our framework due to the discontinuity of the contest success function at the origin.[2]

Proposition A1 also indicates that a CPNE may fail to exist when k exceeds 1/3. This is because contestants' best response may display a discontinuity at a threshold of opponents' aggregate effort, and reaffirms the observation of Dato et al. (2017, Figure 1). Next, we provide two examples to briefly discuss equilibrium existence.

Example A1 *Suppose that Assumption 2 is satisfied and $k \in [0, \frac{1}{2}]$. Consider a contest that involves $N \geq 2$ homogeneous contestants with $v_1 = \cdots = v_N =: v > 0$ for all $i \in \mathcal{N}$. The following statements hold:*

(i) If $k \in [0, \frac{N}{3N-2}]$, then there exists a unique pure-strategy CPNE in which all contestants exert an effort $x^* = \frac{N-1}{N^2}v - \frac{(N-1)(N-2)}{N^3}kv$.

(ii) If $k \in (\frac{N}{3N-2}, \frac{1}{2}]$, then the contest game has no pure-strategy CPNE.

Part (ii) of the above example echoes Proposition 2 of Dato et al. (2017): When players are symmetric, there exists a threshold of the degree of loss aversion above which a CPNE fails to exist.

We now provide another example to illustrate the subtle impact of loss aversion on the existence of CPNE when contestants are heterogeneous.

Example A2 *Suppose that Assumption 2 is satisfied and $k \in [0, \frac{1}{2}]$. Consider a three-player contest with $(v_1, v_2, v_3) = (1, 0.9, 0.8)$. There exist two cutoffs $k_1 \approx 0.3650$ and $k_2 \approx 0.4098$ such that*

(i) For $k \in [0, k_1]$, there exists a unique pure-strategy CPNE in which all three contestants exert a positive amount of effort.

(ii) For $k \in (k_1, k_2)$, the contest game has no pure-strategy CPNE.

(iii) For $k \in [k_2, \frac{1}{2}]$, there exists a unique pure-strategy CPNE, in which contestants 1 and 2 exert a positive amount of effort, whereas contestant 3 remains inactive.

In the same spirit, Figure A1 plots the combination of winning valuations (v_1, v_2, v_3) that lead to a unique CPNE or no CPNE in three-player contests with $k = 0.4$.

[2] To be more specific, once a contestant exerts zero effort, his opponent would sink an infinitesimal amount of effort to win the contest with probability one. This would both increase his material payoff and maximize his gain-loss utility by completely eliminating the underlying uncertainty of his realized payoff.

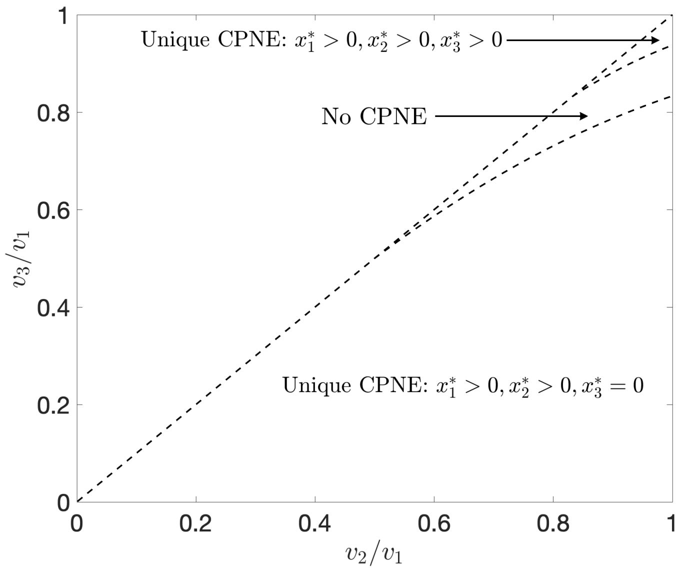


Figure A1: Existence of CPNE in Three-player Contests: $k = 0.4$.

A.2 Contest Design: Contestant Selection

Next, we discuss the impact of loss aversion on contest design. To most cleanly illuminate the implication of loss aversion on contest design, we consider the following simple two-player contest design problem.

A contest designer is running a two-player contest and aims to maximize total effort. There exists an incumbent player whose valuation of winning the prize is normalized to one. The designer can select an opponent, denoted by $\hat{v}$, from a pool of talents/valuations $\mathcal{V} = [0, \infty]$. Denote the opponent's type in the optimal contest by $\hat{v}^*$. The following result can be established:

Proposition A2 *Suppose that Assumption 2 is satisfied and $k \in [0, \frac{1}{2}]$. Fix an arbitrary $\hat{v} \in \mathcal{V}$; there always exists a unique CPNE of the two-player contest game. Moreover, $\hat{v}^* = \infty$ if $k \in [0, \frac{1}{3}]$; and $1 < \hat{v}^* < \infty$ if $k \in (\frac{1}{3}, \frac{1}{2}]$.*

Proof. It is straightforward to verify that there exists a unique CPNE for all $\hat{v} > 0$ from Propositions 1 and A1, and the equilibrium effort profile is given by Corollary 1. Denote the total effort of inviting a contestant with winning valuation $\hat{v}$ by $TE(\hat{v})$. It follows from Corollary 1 that

$$TE(\hat{v}) = \frac{1}{1+\theta(\hat{v})} - \frac{1-\theta(\hat{v})}{\left[1+\theta(\hat{v})\right]^2}k,$$

where

$$\theta(\hat{v}) := \frac{1}{2}\left[\left(\frac{1}{\hat{v}} - 1\right) \times \frac{1+k}{1-k} + \sqrt{\left(\frac{1}{\hat{v}} - 1\right)^2 \times \left(\frac{1+k}{1-k}\right)^2 + \frac{4}{\hat{v}}}\right].$$

Taking the derivative of $TE(\hat{v})$ with respect to $\hat{v}$ yields

$$\frac{dTE}{d\hat{v}} = \frac{d\theta}{d\hat{v}} \times \left[-\frac{1}{(1+\theta)^2} + \frac{3-\theta}{(1+\theta)^3} k \right].$$

Carrying out the algebra, we can obtain

$$\frac{d\theta}{d\hat{v}} = -\frac{1}{2} \times \frac{1}{\hat{v}^2} \times \left[\frac{1+k}{1-k} + \frac{\left(\frac{1}{\hat{v}} - 1\right) \times \left(\frac{1+k}{1-k}\right)^2 + 2}{\sqrt{\left(\frac{1}{\hat{v}} - 1\right)^2 \times \left(\frac{1+k}{1-k}\right)^2 + \frac{4}{\hat{v}}}} \right] < 0, \ \forall \, \hat{v} > 0.$$

Suppose that $k \leq \frac{1}{3}$. Then we have that

$$\frac{dTE}{d\hat{v}} = \frac{d\theta}{d\hat{v}} \times \left[-\frac{1}{(1+\theta)^2} + \frac{3-\theta}{(1+\theta)^3} k \right] \geq -1 \times \frac{d\theta}{d\hat{v}} \times \frac{2}{3}\theta > 0, \ \forall \, \hat{v} > 0,$$

which indicates that $\hat{v}^* = \infty$.

Suppose that $k > \frac{1}{3}$. It can be verified that $\frac{dTE}{d\hat{v}} = 0$ is equivalent to

$$\theta(\hat{v}) = \frac{3k-1}{k+1}.$$

Recall that $\frac{d\theta}{d\hat{v}} < 0$. Moreover, $0 < \frac{3k-1}{k+1} < \frac{1}{3}$ for all $k \in (\frac{1}{3}, \frac{1}{2}]$, $\lim_{\hat{v} \searrow 1} \theta = 2$, and $\lim_{\hat{v} \nearrow \infty} \theta = 0$. Therefore, there exists a unique solution to the above equation and thus $\hat{v}^* \in (1, \infty)$. This completes the proof. ■

By Proposition A2, an effort-maximizing contest designer will select an opponent who is moderately stronger than the incumbent to stimulate the incumbent when contestants are sufficiently loss averse. This result stands in stark contrast to the optimal ability selection problem with standard preferences. Suppose that $k = 0$. In equilibrium, the incumbent exerts effort $\hat{v}/(1+\hat{v})^2$, and the opponent exerts effort $\hat{v}^2/(1+\hat{v})^2$. Simple algebra shows that total effort amounts to $\hat{v}/(1+\hat{v})$, which is strictly increasing in $\hat{v}$. Therefore, the designer would select the strongest player from the pool of talent.

B Concave Impact Function

In this part, we consider a concave impact function and show that our main results continue to hold. In particular, we assume that the impact function takes the form of $f_i(x_i) = (x_i)^r$, with $r \leq 1$ throughout the section. A smaller r implies that winner selection in the contest depends less on their effort input and more on luck.

B.1 Two-player Contests

We begin with a two-player contest and examine the robustness of Proposition 3 in Section 3.1.

Symmetric Players Part (i) of Proposition 3 naturally extends: With symmetric players (i.e.,$v_1 = v_2 =: v$), each wins with a probability 1/2 in equilibrium, which diminishes the marginal effect of a variation in p_i on $p_i(1-p_i)$. As a result, loss aversion does not affect contestants' equilibrium effort, and each player exerts an effort $rv/4$ in equilibrium, as under standard preferences.

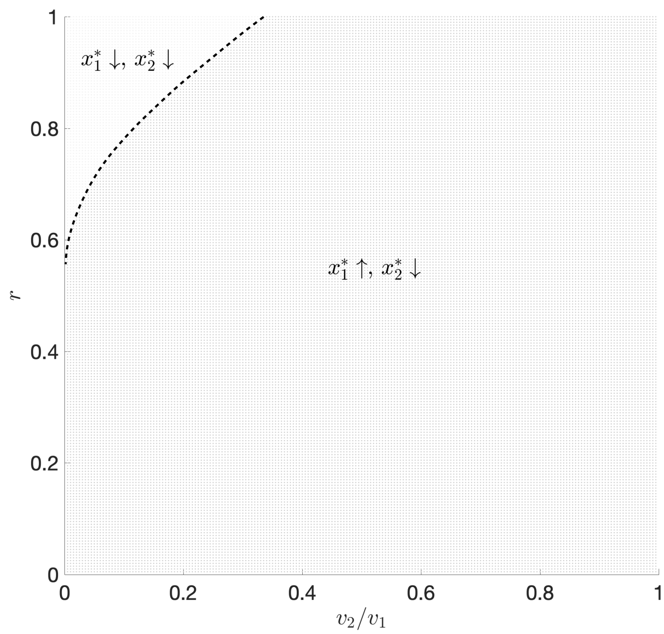


Figure A2: Impact of Reference-dependent Preferences on Incentives in Two-player Contests.

Asymmetric Players Although equilibrium existence and uniqueness can be ensured by Proposition 1, a closed-form profile cannot be obtained in general when players are heterogeneous. We conduct numerical exercises to explore the implications of loss aversion. Figure A2 illustrates the comparison between the equilibrium effort profile when contestants are loss averse ($k = 0.01$) and the counterpart under standard preferences ($k = 0$). The horizontal axis traces $v_2/v_1 \in (0, 1)$ and the vertical axis depicts $r \in (0, 1)$.

In line with Proposition 3(ii), loss aversion always reduces the underdog's equilibrium effort x_2^*, whereas the favorite may either increase or decrease x_1^*. The region of $(v_2/v_1, r)$ to the right (respectively, to the left) of the dashed curve depicts the combinations of $(v_2/v_1, r)$ under which the favorite increases (decreases) his equilibrium effort relative to the case of $k = 0$. Two observations are noteworthy. First, when r is large (e.g., $r = 0.8$), the favorite increases (reduces) his effort when the degree of player heterogeneity is moderate (large). Second, when r is small (e.g., $r = 0.4$), the favorite always increases his effort regardless of the degree of player heterogeneity.

These observations confirm the tension between the uncertainty-reducing effect and the competition effect identified in the baseline setting. A small r amplifies the former and diminishes the latter. Recall that the uncertainty is measured by the term $p_i(1-p_i)$. The uncertainty-reducing effect compels the favorite to step up his effort (i.e., increasing p_1) and the underdog to concede (i.e., decreasing p_2); the favorite is tempted to slack off in response to the less aggressive opponent, which leads to the competition effect. A smaller r implies a noisier winner-selection mechanism and lower marginal return on one's effort. As a result, the favorite has to supply a larger amount of extra effort to achieve a given increase in p_1; conversely, the competition effect is limited because the noise erodes his lead, which prevents him from slacking off. With a smaller r, the competition effect is less likely to outweigh the uncertainty-reducing effect. In contrast, with a larger r, the model converges to our baseline setting and the observation echoes the result in Proposition 3(ii); i.e., the favorite increases his effort under moderate asymmetry.

B.2 Contests with Three or More Contestants

Next, we consider contests with three or more contestants and examine the robustness of Propositions 4 and 5 in Section 3.2.

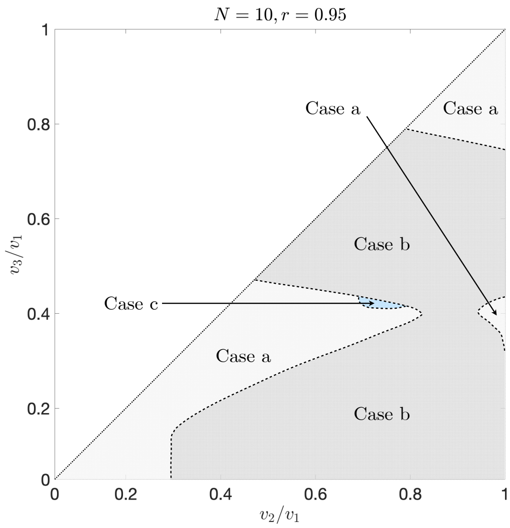
Symmetric Players Suppose that $k \in [0, \frac{1}{3}]$ and the contest involves $N \geq 3$ homogeneous contestants, with $v_1 = \cdots = v_N =: v > 0$ for all $i \in \mathcal{N}$. Simple algebra would verify that all contestants exert an effort

$$x^*(k) = \frac{N-1}{N^2}\left(1 - \frac{N-2}{N}k\right) rv$$

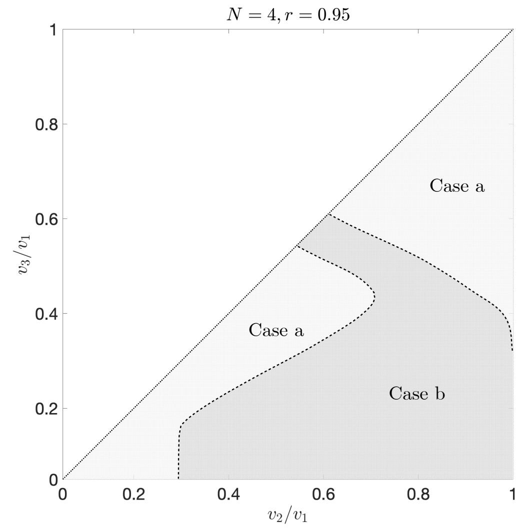
in the unique CPNE, which strictly decreases with k. The comparative statics in Proposition 4 for the case of $r = 1$ are perfectly retained in the case of $r < 1$.

Asymmetric Players The subsequent discussion allows for asymmetric players. In contrast to the case of a linear impact function ($r = 1$), all contestants remain active when the impact function is concave ($r < 1$). Although a formal proof is unavailable, numerical results suggest that the equilibrium outcomes remain largely consistent.

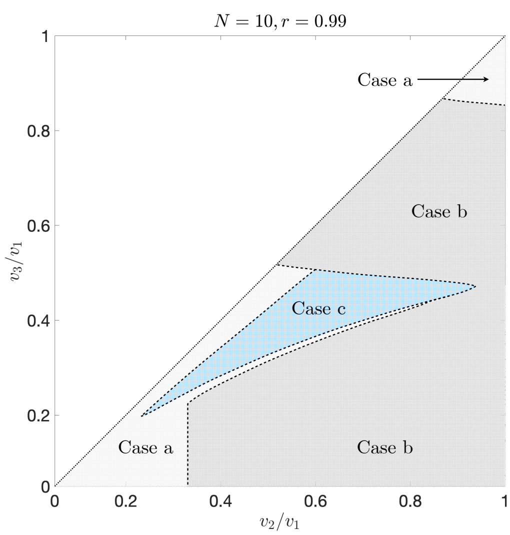
Recall that Proposition 5 predicts three possible cases under linear impact functions. In case (a), loss aversion leads all contestants to reduce their efforts. In case (b), strong contestants step up their efforts, while the weaker do the opposite. Case (c) reports a



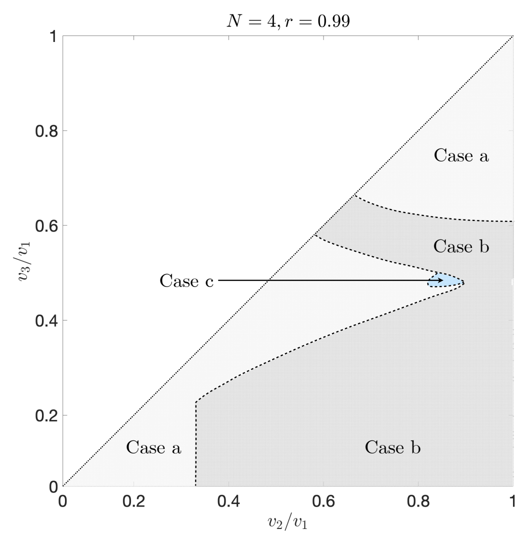
(a) Ten players: $v_1 \geq v_2 \geq v_3 = \cdots = v_{10}$



(b) Four players: $v_1 \geq v_2 \geq v_3 = v_4$



(c) Ten players: $v_1 \geq v_2 \geq v_3 = \cdots = v_{10}$



(d) Four players: $v_1 \geq v_2 \geq v_3 = v_4$

Figure A3: Impact of Reference-dependent Preferences on Player Incentives with a Concave Impact Function.

nonmonotone scenario, in which a set of middle contestants increase their efforts and the rest concede. Figure A3 compares the equilibrium outcomes when contestants are loss averse ($k = 0.01$) with the counterparts under standard preferences ($k = 0$). Three cases may emerge, as in Figure 3.

The left panel [Figures 3(a) and 3(c)] depicts a scenario of 10 contestants ($v_1 \geq v_2 \geq v_3 = \cdots = v_{10}$) and the right panel [Figures 3(b) and 3(d)] represents one of four contestants ($v_1 \geq v_2 \geq v_3 = v_4$). The upper panel assumes $r = 0.95$ and the lower panel depicts the case of $r = 0.99$.

Our discussion focuses on case (c). Comparing the left panel with the right panel, it is straightforward to observe that case (c) is more likely to occur when more weak players are added to the contest, which is consistent with the observations in Section 3.2. A comparison between the upper panel [Figures 3(a) and 3(b)] and the lower panel [Figures 3(c) and 3(d)] implies that case (c) is more likely when r increases.

C PPNE and CPNE

Recall that the PPNE may depart from the CPNE when η is sufficiently large. We now demonstrate in two examples that the main results derived under the equilibrium concept of CPNE in Section 3 remain qualitatively intact under the alternative concept of PPNE.

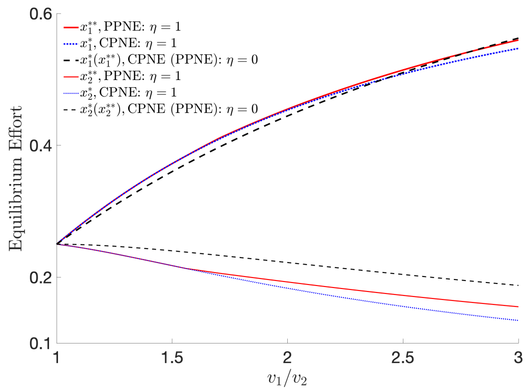
C.1 PPNE and CPNE in Two-Player Contests

Suppose that $f_i(x_i) = x_i$, $N = 2$, and $v_1 \geq v_2$. The unique pure-strategy CPNE coincides with the unique pure-strategy PPNE if and only if

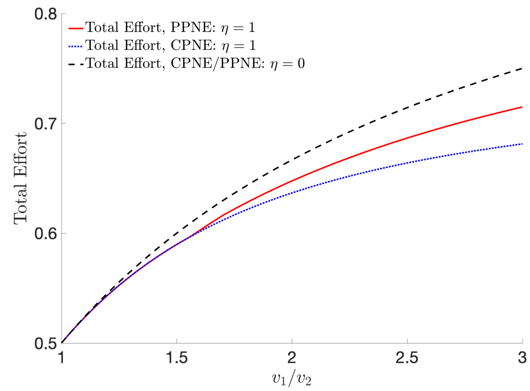
$$\frac{v_1}{v_2} \leq 1 + \frac{\left(\frac{1+\eta\lambda}{\eta\lambda}\right)^2 - 1}{\frac{1+\eta\lambda}{\eta\lambda} \times \frac{1-\eta+\eta\lambda}{1+\eta-\eta\lambda} + 1}.$$

Set $\lambda = 1.25$ and $\eta = 1$, and normalize $v_2 = 1$ without loss of generality. Then the CPNE is the same as the PPNE when $v_1/v_2 \leq 39/25 = 1.56$.

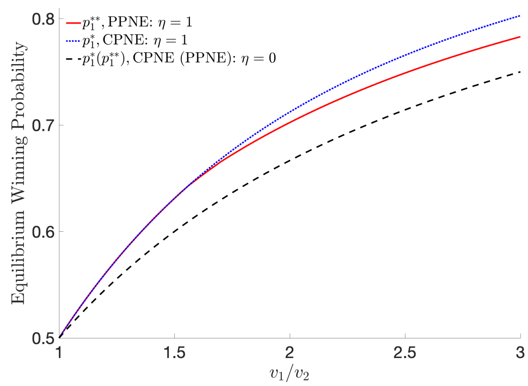
Figure A4 reports contestants' equilibrium effort profile, total effort, and the equilibrium winning probability of the strong contestant in the unique CPNE and PPNE under different levels of v_1/v_2, as well as the counterparts under standard preferences, i.e., $\eta = 0$. Three remarks are in order. First, by Figure 4(a), the weak contestant always exerts a lower effort in the PPNE with the presence of loss aversion than he would under standard preferences. In contrast, loss aversion leads the strong contestant to raise his effort if v_1/v_2 is sufficiently small. These observations affirm the results of Proposition 3 under CPNE. Second, by Figure 4(b), the total effort of loss-averse contestants in the PPNE is always less than that under standard preferences, which echoes the claim of Proposition 7. Finally, by Figure 4(c), loss aversion causes the equilibrium winning odds to bifurcate between the strong contestant and the weak one in the PPNE. Specifically, the strong contestant is more likely to prevail in the competition when η increases from 0 to 1. This observation, again, is consistent with Proposition 8.



(a) Equilibrium Effort



(b) Total Effort



(c) Equilibrium Winning Probability

Figure A4: PPNE vs. CPNE: $(N, v_2, \lambda) = (2, 1, 1.25)$.

C.2 PPNE and CPNE in Contests with Three or More Contestants

We now consider a multi-player contest. Suppose that $f_i(x_i) = x_i$. Set $(N, \lambda) = (8, 1.2)$, and $\boldsymbol{v} \equiv (v_1, v_2, \ldots, v_8) = (2.8, 2.7, \ldots, 2.1)$. The following table reports the equilibrium winning probability profile in the unique CPNE and PPNE when the contestants are loss averse (i.e., $\eta = 1$), as well as that under standard preferences (i.e., $\eta = 0$).

η	Equilibrium concept	p_1^*	p_2^*	p_3^*	p_4^*	p_5^*	p_6^*	p_7^*	p_8^*	Total effort
0	NE/CPNE/PPNE	0.2396	0.2115	0.1811	0.1484	0.1129	0.0743	0.0322	0	2.1291
1	CPNE	0.2879	0.2486	0.2039	0.1522	0.0910	0.0164	0	0	1.8247
1	PPNE	0.2479	0.2177	0.1851	0.1495	0.1106	0.0680	0.0211	0	1.9619

CPNE and PPNE differ for $\eta = 1$. However, the main predictions obtained in Propositions 7 and 8 under the CPNE remain largely intact under the PPNE. Specifically, the equilibrium winning distributions under both CPNE and PPNE become more dispersed when contestants are loss averse, compared with that under standard preferences (i.e., $\eta = 0$). In particular, the strongest (weakest) four contestants have higher (lower) winning odds when $\eta = 1$ than when $\eta = 0$, regardless of the equilibrium concept. Moreover, the total effort of the contest decreases when loss aversion is in place: Under CPNE, it drops from 2.1291 to 1.8247, while under PPNE, it reduces to 1.9619.

D Heterogeneous Loss Aversion

In our baseline model, we assume that contestants are subject to the same level of loss aversion. We now analyze a two-player contest in which contestants may differ in their prize valuations and/or loss aversion. We first characterize the unique CPNE. We then study the impact of loss aversion on players' effort incentives and show that the main results derived in Proposition 3 are robust.

Consider a two-player contest and suppose that the prize valuation and loss aversion of player $i \in \{1, 2\}$ are $v_i > 0$ and $k_i \in [0, \frac{1}{3}]$, respectively, with $v_1 \geq v_2 > 0$. We first characterize the CPNE of the game. Denote by $\big(x_1^*(k_1, k_2), x_2^*(k_1, k_2)\big)$ the equilibrium effort profile. We establish the following result in parallel to that in Corollary 1.

Proposition A3 *Suppose that Assumption 2 is satisfied, $k_1, k_2 \in [0, \frac{1}{3}]$, and $N = 2$. The equilibrium effort pair $\big(x_1^*(k_1, k_2), x_2^*(k_1, k_2)\big)$ is given by*

$$x_1^*(k_1, k_2) = \frac{\Theta}{(1+\Theta)^2} v_1 - \frac{\Theta(1-\Theta)}{(1+\Theta)^3} k_1 v_1, \tag{A1}$$

and

$$x_2^*(k_1, k_2) = \frac{1}{(1+\Theta)^2} v_1 - \frac{1-\Theta}{(1+\Theta)^3} k_1 v_1, \tag{A2}$$

where

$$\Theta = \frac{1}{2}\left[\frac{v_1}{v_2} \times \frac{1+k_1}{1-k_2} - \frac{1+k_2}{1-k_2} + \sqrt{\left(\frac{v_1}{v_2} \times \frac{1+k_1}{1-k_2} - \frac{1+k_2}{1-k_2}\right)^2 + \frac{4v_1}{v_2} \times \frac{1-k_1}{1-k_2}}\,\right]. \tag{A3}$$

Proof. The proof is similar to that of Corollary 1. It follows from the first-order conditions $\left.\frac{\partial \widehat{U}_1(x_1, x_2^*)}{\partial x_1}\right|_{x_1 = x_1^*} = 0$ and $\left.\frac{\partial \widehat{U}_2(x_2, x_1^*)}{\partial x_2}\right|_{x_2 = x_2^*} = 0$ that

$$\frac{x_2^*}{(x_1^* + x_2^*)^2} v_1 - \frac{x_2^*(x_2^* - x_1^*)}{(x_1^* + x_2^*)^3} k_1 v_1 = 1, \tag{A4}$$

and

$$\frac{x_1^*}{(x_1^*+x_2^*)^2}v_2 - \frac{x_1^*(x_1^*-x_2^*)}{(x_1^*+x_2^*)^3}k_2 v_2 = 1. \tag{A5}$$

Let $\Theta := x_1^*/x_2^*$. The above first-order conditions can be rewritten as

$$\frac{1}{1+\Theta}v_1 - \frac{1-\Theta}{(1+\Theta)^2}k_1 v_1 = x_1^* + x_2^*,$$

and

$$\frac{\Theta}{1+\Theta}v_2 - \frac{\Theta(\Theta-1)}{(1+\Theta)^2}k_2 v_2 = x_1^* + x_2^*.$$

Combining the above two equations yields

$$(1-k_2)\Theta^2 - \left[\frac{v_1}{v_2}(1+k_1) - (1+k_2)\right]\Theta - \frac{v_1}{v_2}(1-k_1) = 0. \tag{A6}$$

Solving for Θ, we can obtain that

$$\Theta = \frac{1}{2}\left[\frac{v_1}{v_2}\times\frac{1+k_1}{1-k_2} - \frac{1+k_2}{1-k_2} + \sqrt{\left(\frac{v_1}{v_2}\times\frac{1+k_1}{1-k_2} - \frac{1+k_2}{1-k_2}\right)^2 + \frac{4v_1}{v_2}\times\frac{1-k_1}{1-k_2}}\right].$$

Substituting the above expression and $\Theta \equiv x_1^*/x_2^*$ into (A4) and (A5), we can solve for $x_1^*(k_1,k_2)$ and $x_2^*(k_1,k_2)$ as specified in (A1) and (A2). ■

Proposition A3 allows us to carry out the comparative statics of players' equilibrium efforts with respect to loss aversion. To proceed, we parameterize (k_1,k_2) such that $k_1 = k$ and $k_2 = \alpha k$, with $\alpha \in (0,\infty)$. We write the equilibrium effort profile $\big(x_1^*(k_1,k_2), x_2^*(k_1,k_2)\big)$ established in Proposition A3 as $\big(x_1^*(k), x_2^*(k)\big)$ with slight abuse of notation. We fix the degree of heterogeneity between the two contestants—i.e., α—and examine how their equilibrium efforts vary with the general level of loss aversion, i.e., k. The following result similar to Proposition 3 can be obtained.

Proposition A4 *Suppose that Assumption 2 is satisfied and $N = 2$. The following statements hold:*

(i) If $v_1 = v_2 =: v$, then $x_1^*(k) = x_2^*(k) = \frac{1}{4}v$ and hence $\frac{dx_1^*}{dk}\big|_{k=0} = \frac{dx_2^*}{dk}\big|_{k=0} = 0$.

(ii) If $v_1 > v_2$, then $\frac{dx_2^*}{dk}\big|_{k=0} < 0$. Moreover, $\frac{dx_1^*}{dk}\big|_{k=0} > 0$ if and only if $\frac{v_1}{v_2} < 1 + \frac{2}{\alpha}$.

Proof. Part (i) of the proposition is obvious and it remains to prove part (ii). Let $\ell := v_1/v_2 > 1$. Equation (A6) can be written as

$$(1-\alpha k)\Theta^2 - \big[\ell(1+k) - (1+\alpha k)\big]\Theta - \ell(1-k) = 0.$$

In what follows, we add k to Θ to emphasize that Θ depends on k. It follows from the above equation and the implicit function theorem that

$$\frac{d\Theta(k)}{dk} = \frac{\alpha\left[\Theta(k)\right]^2 + (\ell-\alpha)\ell - \ell}{2(1-\alpha k)\Theta(k) - [\ell(1+k) - (1+\alpha k)]}.$$

Further, we can obtain $\Theta(0) = \ell$ from (A3). Therefore,

$$\left.\frac{d\Theta(k)}{dk}\right|_{k=0} = (1+\alpha) \times \frac{\ell(\ell-1)}{\ell+1} > 0. \tag{A7}$$

Differentiating $x_1^*(k)$ in (A1) with respect to k yields

$$\begin{aligned}\frac{dx_1^*(k)}{dk} =& \frac{1-\Theta(k)}{\left[1+\Theta(k)\right]^3} \times \frac{d\Theta(k)}{dk} \times v_1 - \left[\frac{1-2\Theta(k)}{\left[1+\Theta(k)\right]^3} - \frac{3\Theta(k)\left[1-\Theta(k)\right]}{\left[1+\Theta(k)\right]^4}\right] \times \frac{d\Theta(k)}{dk} \times kv_1 \\ &- \frac{\Theta(k)\left[1-\Theta(k)\right]}{(1+\Theta)^3} \times v_1.\end{aligned}$$

From the above equation and (A7), we can obtain that

$$\left.\frac{dx_1^*(k)}{dk}\right|_{k=0} \gtrless 0 \Leftrightarrow \ell \lesseqgtr 1 + \frac{2}{\alpha}.$$

Similarly, we can show that

$$\left.\frac{dx_2^*(k)}{dk}\right|_{k=0} = -\frac{(\ell-1)[(1+2\alpha)\ell - 1]}{(1+\ell)^4} \times v_1 < 0.$$

This concludes the proof. ■

In conclusion, the predictions obtained under heterogeneous loss aversion do not qualitatively depart from those obtained in the baseline model.

References

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