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Centralized versus decentralized contests with risk-averse players[☆]

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ABSTRACT

We compare the total effort of centralized single-prize contests and decentralized single-prize contests, assuming risk-averse contestants. Decentralized contests outperform centralized contests when contestants are prudent and sufficiently risk averse. However, decentralized contests are never optimal if the designer can arrange centralized multi-prize contests. Contestants' welfare is also discussed.

1. Introduction

Numerous competitive activities resemble a contest in which contestants exert nonrefundable effort to vie for limited prizes. In many situations, a contest designer can strategically vary the number of participants and/or the value of the prize to influence contestants' incentives. The following question naturally arises: Should the designer arrange a centralized winner-take-all contest, in which all contestants compete for a single prize, or should the designer split her prize purse and divide contestants into several independent subcontests?

This question has been investigated by Wärneryd (2001), Fu and Lu (2009), and Beviá and Corchón (2015a). In a Tullock contest framework, Wärneryd (2001, Proposition 1) and Beviá and Corchón (2015a, Proposition 2) show that replicating a contest always increases contestants' effort incentives, or equivalently, a centralized contest outperforms a decentralized contest in terms of total effort.[1] Fu and Lu (2009) further show that a grand contest induces more effort than any split version of the contest. To the best of our knowledge, the literature has assumed that players are risk neutral. Investment in contests, however, is risky and often causes inherent uncertainty in payoffs.[2] Motivated by this observation, we relax the assumption of risk neutrality to capture the gambling nature of contests and explore the impact of contestants' risk attitude on the effort-maximizing contest arrangement.

Specifically, we assume that contestants are risk averse and prudent.[3] Our main finding can be summarized as follows. Section 3 identifies two effects caused separately by risk aversion and prudence that influence contestants' effort incentives, and provides sufficient conditions under which a centralized/decentralized contest generates more total effort (Propositions 1 and 2). Section 4 adopts a multiple-winner nested Tullock contest, as suggested by Clark and Riis (1996, 1998), and generalizes the model to allow for an alternative contest arrangement that awards multiple prizes in a grand contest. We show that the beauty of "bigness" is restored, in the sense that a decentralized

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1 Assuming a contest success function (CSF) that generalizes the Tullock CSF, Beviá and Corchón (2015a) provide conditions under which a decentralized contest outperforms a centralized contest. See Section 3.3 for more discussion.

2 See Fu and Wu (2019) for a recent survey of theoretical studies on contests.

3 An economic agent's preference is said to exhibit prudence if his marginal utility function is convex in consumption.

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single-prize contest is never optimal: The optimum is either a centralized contest with a single prize or one with multiple prizes (Proposition 4). Section 5 compares contestants' welfare across different contest arrangements. Although the effort comparison depends on contestants' risk attitude, we find that the welfare comparison is unambitious and does not depart from that obtained under risk neutrality (Proposition 5) when contestants exhibit constant absolute risk aversion (CARA).

2. The model

Consider a contest with $N = n \cdot k$ homogeneous contestants and $k \geq 2$ identical prizes. The valuation of each prize is $v > 0$ to all contestants. In the baseline model, we assume that the contest designer can allocate the prizes through either a *decentralized* contest or a *centralized* contest. In a decentralized contest, contestants are divided into k independent identical groups, and $n \geq 2$ players in each group compete to vie for a prize with valuations of v. In a centralized contest, N contestants compete against all others and the winner receives a grand prize, which is the sum of all k prizes. In other words, a centralized contest is the aggregation of k identical decentralized subcontests.

Winner-selection mechanism. The prize is allocated through a standard winner-take-all Tullock contest. Fix a group of players Ω and contestants' effort profile $\boldsymbol{x} := (x_1, \ldots, x_{|\Omega|})$. Player i's probability of winning the prize is given by

$$p_i(\boldsymbol{x}; \Omega) = \begin{cases} \dfrac{(x_i)^r}{\sum_{j\in\Omega}(x_j)^r} & \text{, if } \sum_{j\in\Omega}(x_j)^r \neq 0, \\ \dfrac{1}{|\Omega|} & \text{, if } \sum_{j\in\Omega}(x_j)^r = 0, \end{cases} \tag{1}$$

where $(x_i)^r$ is referred to as the impact function in the contest literature. To guarantee the existence of a pure-strategy equilibrium, we assume a concave impact function, i.e., $r \in (0, 1]$.

Contestants' preference. Each contestant is endowed with initial wealth $w > 0$. The contestants' utility function $u(\cdot)$ is strictly increasing in consumption, twice differentiable, and weakly concave, i.e., $u'' \leq 0 < u'$. Effort is costly and reduces a contestant's wealth. Suppose that a contestant i exerts effort $x_i \geq 0$; he wins a prize $\mathcal{V} > 0$[4] with some probability P_i and receives nothing with probability $1 - P_i$. Then his expected utility, which we denote by π_i, is given by

$$\pi_i := P_i \cdot u(w + \mathcal{V} - x_i) + (1 - P_i) \cdot u(w - x_i). \tag{2}$$

We further impose the following condition on contestants' preference throughout the paper:

Assumption 1 (*NIARA Preferences*)**.** Contestants' utility function exhibits nonincreasing absolute risk aversion (NIARA), i.e., $-u''(c)/u'(c)$ is nonincreasing in c.

The NIARA condition is satisfied by a wide array of utility functions, such as constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility functions. It is noteworthy that NIARA implies that contestants are prudent, i.e., $u''' \geq 0$.

3. Decentralized vs. centralized single-prize contests

3.1. Equilibrium analysis

Equilibrium under decentralization. We first characterize the equilibrium in a decentralized contest. We focus on symmetric pure-strategy equilibria in which all n contestants in the group exert the same amount of effort. Suppose that all players other than one indicative player i place the same bid x_d in equilibrium, where we use the subscript d to indicate "decentralized contest". Player i chooses x_i to maximize his expected payoff

$$\max_{x_i \geq 0} \pi_i(x_i, x_d) := \frac{(x_i)^r}{(x_i)^r + (n-1)(x_d)^r} \cdot u(w + v - x_i) + \frac{(n-1)(x_d)^r}{(x_i)^r + (n-1)(x_d)^r} \cdot u(w - x_i).$$

The first-order condition with respect to x_i leads to

$$\frac{(n-1)(x_d)^r}{(x_i)^r + (n-1)(x_d)^r} \cdot \frac{r \cdot (x_i)^{r-1}}{(x_i)^r + (n-1)(x_d)^r} \cdot [u(w + v - x_i) - u(w - x_i)] = \frac{(x_i)^r}{(x_i)^r + (n-1)(x_d)^r} \cdot u'(w + v - x_i) + \frac{(n-1)(x_d)^r}{(x_i)^r + (n-1)(x_d)^r} \cdot u'(w - x_i).$$

Imposing the symmetric condition $x_i = x_d$ to the above first-order condition gives

$$\underbrace{\frac{r}{x_d}\overbrace{\left[\left(1 - \frac{1}{n}\right)\frac{1}{n}\right]}^{\mathcal{A}_d} \times \overbrace{\Big[u(w + v - x_d) - u(w - x_d)\Big]}^{\mathcal{B}_d}}_{\text{marginal benefit of effort under decentralization}} = \underbrace{\overbrace{\left[\frac{1}{n}u'(w + v - x_d) + \left(1 - \frac{1}{n}\right)u'(w - x_d)\right]}^{\mathcal{C}_d}}_{\text{marginal cost of effort under decentralization}}. \tag{3}$$

By Yamazaki (2009), the above candidate symmetric equilibrium constitutes the unique pure-strategy equilibrium under Assumption 1.

Before we proceed, it is useful to take a closer look at the equilibrium first-order condition (3). Contestants' marginal benefit of effort depends on the number of players in the competition, which is captured by the term $\mathcal{A}_d$, and the difference in utility between winning and losing, which depends on contestants' utility function $u(\cdot)$ and is captured by the term $\mathcal{B}_d$. Note that contestants' risk aversion also influences the marginal cost of effort. With risk neutrality in place, contestants' marginal cost of effort degenerates to a constant. When contestants are risk averse, however, their marginal cost depends also on the shape of their marginal utility function $u'(\cdot)$, which is captured by the term $\mathcal{C}_d$.

Equilibrium under centralization. The equilibrium analysis in a centralized contest is similar. Denote the equilibrium effort in a centralized contest by x_c; again, we use the subscript c to indicate "centralized contest". It can be shown that x_c is the unique solution to the following condition:

$$\underbrace{\frac{r}{x_c}\overbrace{\left[\left(1 - \frac{1}{kn}\right)\frac{1}{kn}\right]}^{\mathcal{A}_c} \times \overbrace{\Big[u(w + kv - x_c) - u(w - x_c)\Big]}^{\mathcal{B}_c}}_{\text{marginal benefit of effort under centralization}} = \underbrace{\overbrace{\left[\frac{1}{kn}u'(w + kv - x_c) + \left(1 - \frac{1}{kn}\right)u'(w - x_c)\right]}^{\mathcal{C}_c}}_{\text{marginal cost of effort under centralization}}. \tag{4}$$

The above discussions are summarized in the following lemma:

[4] Clearly, $\mathcal{V} = v$ under decentralization and $\mathcal{V} = kv$ under centralization.

Lemma 1 (*Equilibrium in Decentralized/centralized Contests*)**.** *Suppose that Assumption 1 is satisfied. Then there exists a unique symmetric pure-strategy equilibrium in a decentralized and a centralized contest, in which each contestant's equilibrium effort is the solution to Eqs. (3) and (4), respectively.*

3.2. Optimal contest arrangement: Decentralization vs. centralization

We are now ready to explore the optimal contest arrangement based on the equilibrium characterization established in Lemma 1. Wärneryd (2001) shows that a centralized contest outperforms a decentralized contest if the contestants are risk neutral. Next, we derive sufficient conditions under which this result continues to hold. For notational convenience, let us define

$$\ell := \sup_{w-v/n\leq c\leq w+kv} u'''(c).$$

Proposition 1 (*Optimality of Centralized Contests*)**.** *Suppose that Assumption 1 is satisfied. Then a centralized contest outperforms a decentralized contest, i.e.,* $x_c > x_d$, *if*

$$-\frac{u''(c)}{u'(c)}+\frac{vk}{3}\left(1+\frac{3}{r}\right)\frac{\ell}{u'(c)}<\frac{2}{vnk^2}, \text{ for all } w\leq c\leq w+kv. \quad (5)$$

Note that the above condition is more likely to be satisfied when (i) the degree of contestants' absolute risk aversion (i.e., $-u''(c)/u'(c)$) remains moderate and (ii) $\ell/u'(c)$ are sufficiently small. Clearly, this occurs when $u'(c)$ decreases gradually, i.e., when the contestant's insurance motive in a contest is not excessively strong.

Proposition 1 extends the boundary for the dominance result of centralized contests in Wärneryd (2001, Proposition 1) and Beviá and Corchón (2015a, Proposition 2), as it allows for weak risk aversion and/or weak prudence. To see this, note that the above condition automatically holds when contestants are risk neutral: The left-hand side of the condition boils down to zero, whereas the right-hand side is strictly positive.

Next, we provide sufficient conditions under which individual/aggregate effort is larger in the decentralized contests.

Proposition 2 (*Optimality of Decentralized Contests*)**.** *Suppose that Assumption 1 is satisfied and* $n \geq 3$. *Then a decentralized contest generates more individual/total effort than a centralized contest if* $-\frac{u''(c)}{u'(c)} > \frac{2}{v(n-2)}$ *for all* $w \leq c \leq w+kv$.

Aggregating several decentralized contests into a centralized contest triggers two opposing effects when contestants are risk neutral (Beviá and Corchón, 2015a). On the one hand, the marginal benefit of effort is reduced due to the increasing number of competitors a contestant faces. This effect is reflected by the fact that $\mathcal{A}_c < \mathcal{A}_d$, and tends to attenuate their effort incentive in the equilibrium (*competition effect*). On the other hand, the prize money is larger in a centralized contest upon winning, which creates a larger prize spread. This effect is rendered by the fact that $\mathcal{B}_c > \mathcal{B}_d$, and increases contestants' effort incentive (*prize effect*). The former competition effect favors decentralization, while the latter effect favors centralization. In a Tullock contest with risk-neutral players, the former competition effect is dominated by the latter prize effect. As a result, centralization is preferred over decentralization by an effort-maximizing contest designer.

When contestants are risk averse and prudent, two additional effects would influence the designer's optimal choice of contest arrangement. On the one hand, the concavity of the utility function tends to discount the extra utility gain from the increase of prize money, and thus trivializes the prize effect. This favors decentralization. On the other hand, when contestants are prudent, they are averse to downward risk, i.e., losing the contest. Because contestants' probability of losing the contest and the amount they lose is smaller under decentralization, contestants' marginal cost of effort under decentralization is lower than under centralization ($\mathcal{C}_d < \mathcal{C}_c$), holding the effort level fixed. This again favors decentralization. Therefore, the equilibrium effort under decentralization is higher when contestants are prudent and sufficiently risk averse, as predicted in Proposition 2.

3.3. Robustness

Next, we generalize the contest success function (1) and show that the main result we derive in Proposition 2 remains qualitatively unchanged. Specifically, we follow Beviá and Corchón (2015a,b) and consider the following generalized contest success function (CSF):

$$p_i(\boldsymbol{x};\Omega)=\begin{cases}\dfrac{1-\beta(1-s)}{|\Omega|}+\beta\dfrac{(x_i)^r-\frac{s}{|\Omega|-1}\sum_{j\in\Omega,j\neq i}(x_j)^r}{\sum_{j\in\Omega}(x_j)^r} & \text{, if } \sum_{j\in\Omega}(x_j)^r\neq 0,\\ \dfrac{1}{|\Omega|} & \text{, if } \sum_{j\in\Omega}(x_j)^r=0,\end{cases}$$

with $0<\beta\leq(|\Omega|-1)/(|\Omega|-1+s)$ and $s>1-|\Omega|$.[5] Note that the above CSF degenerates to (1) when $(\beta, s) = (1, 0)$.

Beviá and Corchón (2015a) show that the effort comparison between a centralized and decentralized contest hinges on the parameter s when contestants are risk neutral.

Lemma 2 (*Beviá and Corchón, 2015a*)**.** *A decentralized contest generates more individual/total effort than a centralized contest if and only if* $s > 1$ *when contestants are risk neutral.*

By Lemma 2, the competition effect dominates the prize effect when $s > 1$, and decentralization is preferred. Intuitively, introducing risk aversion would strengthen the dominance of decentralization in this case. When $s < 1$, we can show that Proposition 2 is robust, given that s is not excessively small.

Proposition 3 (*Optimality of Decentralized Contests with a Generalized CSF*)**.** *Suppose that* $s > 1-\frac{n}{2}$ *and a symmetric pure-strategy equilibrium exists. Then a decentralized contest generates more individual/total effort than a centralized contest if* $-\frac{u''(c)}{u'(c)} > \frac{2(1-s)}{v[n-2(1-s)]}$ *for all* $w \leq c \leq w+kv$.

Suppose that contestants exhibit constant absolute risk aversion (CARA) and the utility function takes the form $u(c) = 1 - \exp(-\alpha c)$, with $\alpha > 0$. It can be verified that a symmetric pure-strategy equilibrium under both decentralization and centralization exists if $\beta \leq (n-1)/(n-1+s)$. Fig. 1 depicts the optimal contest arrangement in the (α, s) space. When $s \geq 1$, decentralization is preferred regardless of contestants' degree of risk aversion. When $s \leq -(n-1)(kn-1)/[(k+1)n-1]$, centralization is preferred independent of α. When s falls in the intermediate range, there exists a threshold of risk aversion α above which (respectively, below which) decentralization (respectively, centralization) outperforms.

4. Centralized contests with multiple prizes

Thus far, we assume that contestants compete to vie for a single prize. In this section, we extend the model to allow for multiple prizes. To this end, we follow Fu and Lu (2009) and

[5] The conditions $0<\beta\leq(|\Omega|-1)/(|\Omega|-1+s)$ and $s>1-|\Omega|$ ensure that (i) $p_i(\boldsymbol{x};\Omega)$ is strictly increasing in contestant i's effort x_i, and (ii) $p_i(\boldsymbol{x};\Omega)\geq 0$ at $x_i = 0$.

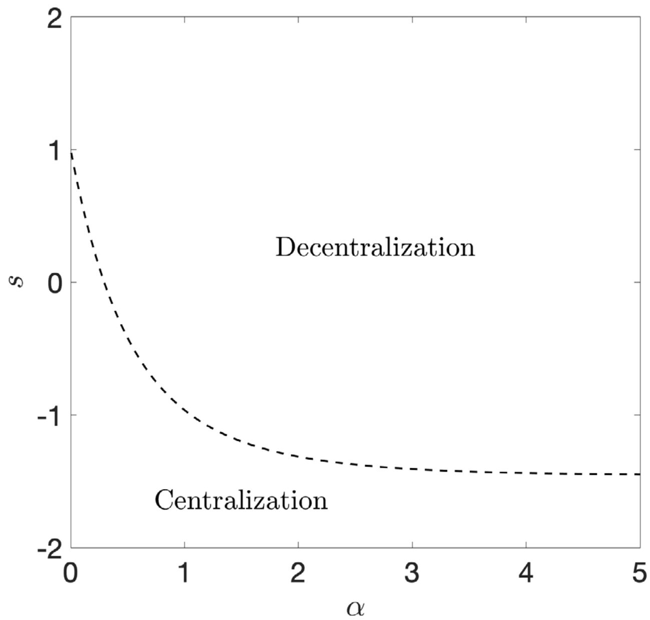


Fig. 1. Centralization vs. decentralization with a generalized CSF and CARA preferences: $(n, k, v, w, r, \beta) = (3, 3, 1, 0, 1, 0.5)$.

assume that the designer can adopt a centralized contest to allocate k identical prizes through a multiple-winner nested Tullock contest, as suggested by Clark and Riis (1996, 1998): Fixing contestants' effort profile $\boldsymbol{x} \equiv (x_1, \ldots, x_N)$, k identical prizes are to be given away sequentially; the recipients of the prizes are selected by k consecutive draws; once a contestant is selected to win a prize, he is immediately removed from the pool of candidates up for the next draw. The process is repeated until all k prizes have been distributed. More formally, denote Ω_q as the set of remaining contestants for the qth draw, where $q \in \{1, \ldots, k\}$. Then the conditional probability of a contestant $i \in \Omega_q$ winning the sth prize is specified in Eq. (1).

4.1. Equilibrium analysis in centralized multi-prize contests

Fu et al. (2019) prove the existence and uniqueness of a symmetric pure-strategy equilibrium in a centralized multi-prize contest when Assumption 1 is satisfied.[6] Denote the equilibrium effort by x_m, where we use the subscript m to indicate "multi-prize contests". Standard analysis verifies that x_m is the solution to the following first-order condition:

$$\underbrace{\frac{r}{x_m}\overbrace{\left[\left(1-\frac{1}{n}\right)\lambda\right]}^{\mathcal{A}_m} \times \overbrace{\left[u\left(w+v-x_m\right)-u(w-x_m)\right]}^{\mathcal{B}_m}}_{\text{marginal benefit of effort in a multi-prize contest}}$$

$$= \underbrace{\overbrace{\left[\frac{1}{n}u'\left(w+v-x_m\right)+\left(1-\frac{1}{n}\right)u'(w-x_m)\right]}^{\mathcal{C}_m}}_{\text{marginal cost of effort in a multi-prize contest}}. \tag{6}$$

where $\lambda := \sum_{j=0}^{k-1} \frac{1}{kn-j} > \frac{1}{n}$. The above discussions are summarized in the following lemma:

Lemma 3 (*Equilibrium in Centralized Multi-prize Contests*)**.** *Suppose that Assumption 1 is satisfied. Then there exists a unique pure-strategy equilibrium in a centralized multi-prize contest, in which each contestant's equilibrium effort is the solution to Eq. (6).*

4.2. Optimal contest arrangement, revisited

Proposition 4 (*Effort Comparison and Optimal Contest Arrangement*)**.** *Suppose that Assumption 1 is satisfied. Then the following statements hold:*

i. *A decentralized single-prize contest generates strictly lower individual effort than a centralized multi-prize contest, i.e., $x_m > x_d$.*
ii. *If $-\frac{u''(c)}{u'(c)} + \frac{vk}{3}\left(1+\frac{3}{r}\right)\frac{\ell}{u'(c)} < \frac{1}{vnk^3}$ for all $c \in [w, w+kv]$, then $x_m < x_c$; if $-\frac{u''(c)}{u'(c)} > \frac{2}{v(n-2)}$ for all $c \in [w, w+kv]$, then $x_m > x_c$.*

Part (i) of Proposition 4 states that a decentralized single-prize contest is always outperformed by a centralized multi-prize contest in terms of total effort. Therefore, the optimal contest arrangement must be either a centralized single-prize contest or a centralized multi-prize contest, which revives the beauty of "bigness" with the presence of risk aversion. To understand the result, it is useful to reexamine first-order conditions (3) and (6). First, a decentralized single-prize contest and a centralized multi-prize contest create the same prize spread, and thus lead to identical utility gain from winning a prize ($\mathcal{B}_d = \mathcal{B}_m$). Second, the probabilities of not receiving a prize under both contest arrangements coincide, indicating that the marginal costs are the same for an arbitrary effort level (i.e., $\mathcal{C}_d = \mathcal{C}_m$). Third, a contestant's effort in a multi-prize contest yields additional chances to win a prize even if he missed the first prize (i.e., $\mathcal{A}_d < \mathcal{A}_m$). Therefore, competition tends to be fiercer in a multi-prize contest due to the higher marginal impact of effort on contestants' winning probability. Part (i) of Proposition 4 follows immediately from the above part-to-part comparison of Eqs. (3) and (6).

Part (ii) Proposition 4 provides sufficient conditions under which a single-prize or a multi-prize centralized contest is optimal. With CARA preferences, we can obtain a closed-form expression of the equilibrium effort under each contest arrangement [see Eqs. (9)–(11)], and further obtain the following result [see also Fig. 2(a)].

Example 1 (*Effort Comparison with CARA Preferences*)**.** Suppose that the utility function takes the form $u(c) = 1 - \exp(-\alpha c)$, with $\alpha > 0$. Then there exist two thresholds of player's risk aversion $\overline{\alpha}_1$ and $\overline{\alpha}_2$, with $\overline{\alpha}_1 < \overline{\alpha}_2$, such that:

i. For $\alpha \in (0, \overline{\alpha}_1)$, $x_d < x_m < x_c$;
ii. For $\alpha \in (\overline{\alpha}_1, \overline{\alpha}_2)$, $x_d < x_c < x_m$;
iii. For $\alpha \in (\overline{\alpha}_2, \infty)$, $x_c < x_d < x_m$.

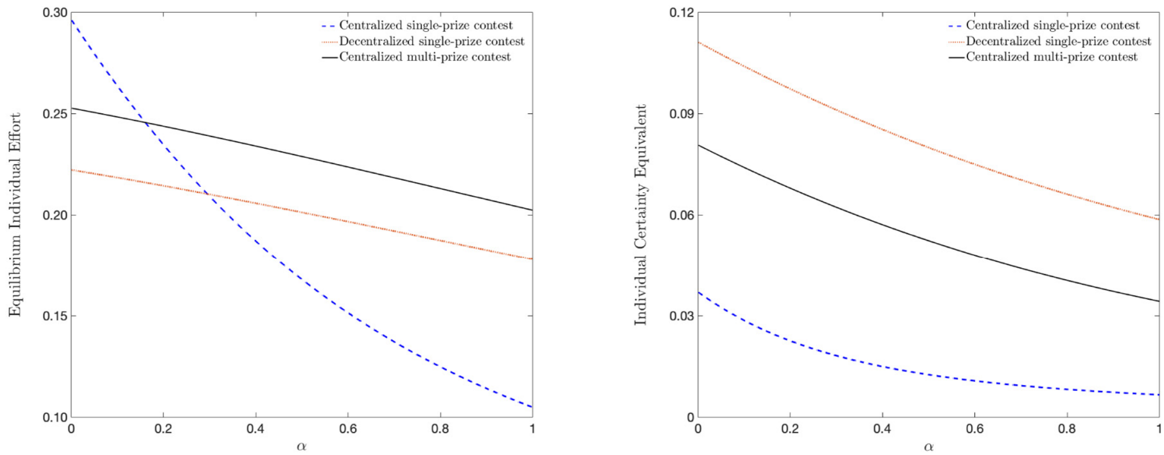
5. Welfare comparison with CARA preferences

In this section, we consider contestants' welfare. The welfare comparison with risk-neutral players is obvious, because each contestant's payoff is simply $v/n - x_\theta$, where $\theta \in \{c, d, m\}$. Simple algebra gives

$$\frac{v}{n} - x_d > \frac{v}{n} - x_m > \frac{v}{n} - x_c.$$

Interestingly, although the effort comparison may vary with the presence of risk aversion, as stated in Propositions 2 and 4, the above welfare ranking under risk neutrality seems to extend to contests with risk-averse players.

[6] Fu et al. (2019) allow for a decreasing prize series and investigate the optimal prize allocation.



(a) Individual Effort Comparison

(b) Individual Welfare Comparison

Fig. 2. Contest performance under different levels of risk aversion α: $(n, k, v, w, r) = (3, 3, 1, 0, 1)$.

Next, we compare contestants' welfare across the three contest arrangements within the class of CARA preferences. We measure a contestant's welfare by his certainty equivalent (CE) from participating in the contest. Suppose that contestant i exerts effort x_i and has a probability P_i of winning a prize $\mathcal{V}$, as specified in Eq. (2). Then his certainty equivalent, which we denote by CE, is given by

$$CE = u^{-1}\left(P_i \cdot u\left(w + \mathcal{V} - x_i\right) + (1 - P_i) \cdot u\left(w - x_i\right)\right). \tag{7}$$

Denote contestants' equilibrium certainty equivalent under a decentralized single-prize contest, a centralized single-prize contest, and a centralized multi-prize contest, by CE_d, CE_c, and CE_m, respectively. The following result can be obtained:

Proposition 5 (*Welfare Comparison with CARA Preferences*)**.** *Suppose that the utility function takes the form $u(c) = 1 - \exp(-\alpha c)$, with $\alpha > 0$. Then contestants' welfare under the three contest arrangements can be ranked as follows:*

$$CE_d > CE_m > CE_c.$$

Proposition 5 establishes an unambiguous welfare comparison with CARA preferences [see Fig. 2(b)], which is the same as that with risk-neutral contestants. Simulations indicate that the ranking is preserved for general increasing and concave utility functions (e.g., CRRA preference).[7]

6. Concluding remarks

In this paper, we assume risk-averse contestants and investigate the optimal contest arrangement by comparing the contest performance in (i) a decentralized single-prize contest, (ii) a centralized single-prize contest, and (iii) a centralized multi-prize contest. We show that a decentralized single-prize contest is never optimal in generating total effort, whereas it may lead to the highest welfare for contestants in terms of certainty equivalent.

[7] We can formally show that $CE_d > CE_m$ hold for general preferences due to the fact that (i) the prize valuations and contestants' equilibrium winning probability in a decentralized contest are equal to those in a centralized multi-prize contest; and (ii) $x_m > x_d$. However, it is difficult to prove $CE_m > CE_c$ for the case $x_m > x_c$, which may occur by part (ii) of Proposition 4.

Appendix. Proofs

Proof of Proposition 1.

Proof. Let $x = g(\tau)$ be the solution of the following equation:

$$\begin{aligned}\mathcal{L}(x,\tau) := &\frac{r}{x}\left(1-\frac{\tau}{n}\right)\frac{\tau}{n}\left[u\left(w+\frac{v}{\tau}-x\right)-u(w-x)\right]\\ &-\left[\frac{\tau}{n}u'\left(w+\frac{v}{\tau}-x\right)+\left(1-\frac{\tau}{n}\right)u'(w-x)\right]=0. \end{aligned}\tag{8}$$

It follows immediately from Eqs. (3) and (4) that $x_d = g(1)$ and $x_c = g(1/k)$. Moreover, we have $x = g(\tau) < v/\tau$. To prove the proposition, it suffices to show that

$$g'(\tau) = -\frac{\partial\mathcal{L}/\partial\tau}{\partial\mathcal{L}/\partial x} < 0,\ \forall\, \tau \in \left[\frac{1}{k}, 1\right].$$

It follows from the existence and uniqueness of equilibria in Lemma 1 that

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial x} =& \frac{\partial^2\pi}{\partial x^2} - \frac{r^2}{x^2}\left(1-\frac{\tau}{n}\right)\frac{\tau}{n}\left(1-\frac{2\tau}{n}\right)\\ &\times\left[u\left(w+\frac{v}{\tau}-x\right)-u(w-x)\right]\\ &+\frac{r}{x}\left(1-\frac{\tau}{n}\right)\frac{\tau}{n}\left[u'\left(w+\frac{v}{\tau}-x\right)-u'(w-x)\right]\\ \le& \frac{\partial^2\pi}{\partial x^2} < 0.\end{aligned}$$

Therefore, it remains to show that $\partial\mathcal{L}/\partial\tau < 0$ for all $\tau \in [\frac{1}{k}, 1]$. By Taylor's formula, there exist $\xi_1, \xi_2 \in (w - x, w + \frac{v}{\tau} - x)$ such that

$$\begin{aligned}&u\left(w+\frac{v}{\tau}-x\right)-u(w-x)\\ &\quad=\frac{v}{\tau}u'\left(w+\frac{v}{\tau}-x\right)-\frac{v^2}{2\tau^2}u''\left(w+\frac{v}{\tau}-x\right)+\frac{v^3}{6\tau^3}u'''(\xi_1),\end{aligned}$$

and

$$u'\left(w+\frac{v}{\tau}-x\right)-u'(w-x)=\frac{v}{\tau}u''\left(w+\frac{v}{\tau}-x\right)-\frac{v^2}{2\tau^2}u'''(\xi_2).$$

The above two equations, together with Eq. (8), yield that

$$\frac{\partial\mathcal{L}}{\partial\tau} = \frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\left[-\frac{v}{\tau}\frac{\tau}{n-2\tau}u'\left(w+\frac{v}{\tau}-x\right)\right.$$

$$-\frac{v^2}{2\tau^2}u''\left(w+\frac{v}{\tau}-x\right)+\frac{v^3}{6\tau^3}u'''(\xi_1)\Bigg]+\frac{v^2}{2n\tau^2}u'''(\xi_2).$$

Note that $(w-x, w+\frac{v}{\tau}-x) \subset (w-v/n, w+kv)$. It follows immediately

$u'''(\xi_1) \leq \ell$, and $u'''(\xi_2) \leq \ell$.

Moreover, (5) implies that

$$u'\left(w+\frac{v}{\tau}-x\right) \geq -\frac{vk^2n}{2}u''\left(w+\frac{v}{\tau}-x\right)+\frac{v^2k^3n}{6}\left(1+\frac{3}{r}\right)\ell,$$

Combining the above conditions, we can obtain that

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau} &\leq \frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\Bigg[-\frac{v}{\tau}\frac{\tau}{n-2\tau}u'\left(w+\frac{v}{\tau}-x\right) \\
&\quad -\frac{v^2}{2\tau^2}u''\left(w+\frac{v}{\tau}-x\right)+\frac{v^3}{6\tau^3}\ell\Bigg]+\frac{v^2}{2n\tau^2}\ell \\
&\leq \frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\Bigg\{-\frac{v}{\tau}\frac{\tau}{n-2\tau}\Bigg[-\frac{vk^2n}{2}u''\left(w+\frac{v}{\tau}-x\right) \\
&\quad +\frac{v^2k^3n}{6}\left(1+\frac{3}{r}\right)\ell\Bigg] \\
&\quad -\frac{v^2}{2\tau^2}u''\left(w+\frac{v}{\tau}-x\right)+\frac{v^3}{6\tau^3}\ell\Bigg\}+\frac{v^2}{2n\tau^2}\ell \\
&= \frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\frac{v^2}{2\tau^2}\left(\frac{k^2\tau^2 n}{n-2\tau}-1\right)u''\left(w+\frac{v}{\tau}-x\right) \\
&\quad +\Bigg[\frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\frac{v^3}{6\tau^3}\left(1-\frac{k^3\tau^3 n}{n-2\tau}\right) \\
&\quad +\frac{v^2}{2nx\tau^2}\left(x-vk^3\tau^2\right)\Bigg]\ell < 0,
\end{aligned}$$

where the last inequality follows from $u'' \leq 0$, $\tau \geq 1/k$, and $x \equiv g(\tau) < v/\tau$. This concludes the proof. ■

Proof of Proposition 2.

Proof. By the same argument as in the proof of Proposition 1, it suffices to show that $\partial\mathcal{L}/\partial\tau > 0$ for all $\tau \in [\frac{1}{k}, 1]$. Carrying the algebra, we have that

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau} &= \frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\Bigg[-\frac{v}{\tau}\frac{\tau}{n-2\tau}u'\left(w+\frac{v}{\tau}-x\right) \\
&\quad -\frac{v^2}{2\tau^2}u''\left(w+\frac{v}{\tau}-x\right)+\frac{v^3}{6\tau^3}u'''(\xi_1)\Bigg]+\frac{v^2}{2n\tau^2}u'''(\xi_2) \\
&\geq \frac{r}{nx}\left(1-\frac{2\tau}{n}\right)\Bigg[-\frac{v}{\tau}\frac{\tau}{n-2\tau}u'\left(w+\frac{v}{\tau}-x\right) \\
&\quad -\frac{v^2}{2\tau^2}u''\left(w+\frac{v}{\tau}-x\right)\Bigg] \\
&= \frac{r}{nx}\frac{v^2}{2\tau^2}\left(1-\frac{2\tau}{n}\right)u'\left(w+\frac{v}{\tau}-x\right) \\
&\quad \times\left[-\frac{u''\left(w+\frac{v}{\tau}-x\right)}{u'\left(w+\frac{v}{\tau}-x\right)}-\frac{2\tau^2}{v(n-2\tau)}\right] > 0,
\end{aligned}$$

where the first inequality follows from $u''' \geq 0$, and the second inequality follows from $-\frac{u''(c)}{u'(c)} > \frac{2}{v(n-2)} \geq \frac{2\tau^2}{v(n-2\tau)}$ for all $c \in [w, w+kv]$. This concludes the proof. ■

Proof of Proposition 3.

Proof. The proof is similar to that of Proposition 2, and is omitted for brevity. ■

Proof of Proposition 4.

Proof. Part (i) of the proposition follows immediately from the comparison of Eqs. (3) and (6) and the fact that $\lambda > 1/n$. The proof of part (ii) is similar to that of Propositions 1 and 2, and is omitted for brevity. ■

Proof of Proposition 5.

Proof. With CARA preferences, x_d, x_c, and x_m can be solved from Eqs. (3), (4), and (6), respectively, as follows:

$$x_d = \frac{r\frac{1}{n}\left(1-\frac{1}{n}\right)\left(1-\exp(-\alpha v)\right)}{\alpha\left[1-\frac{1}{n}\left(1-\exp(-\alpha v)\right)\right]}, \tag{9}$$

$$x_c = \frac{r\frac{1}{kn}\left(1-\frac{1}{kn}\right)\left(1-\exp(-\alpha kv)\right)}{\alpha\left[1-\frac{1}{kn}\left(1-\exp(-\alpha kv)\right)\right]}, \tag{10}$$

and

$$x_m = \frac{r\lambda\left(1-\frac{1}{n}\right)\left(1-\exp(-\alpha v)\right)}{\alpha\left[1-\frac{1}{n}\left(1-\exp(-\alpha v)\right)\right]}, \tag{11}$$

where $\lambda = \sum_{j=0}^{k-1}\frac{1}{kn-j}$. Contestants' certainty equivalent under different contest arrangements can be derived by substituting the above equilibrium efforts into (7), and the ranking in contestants' equilibrium welfare in Proposition 5 follows immediately. This concludes the proof. ■

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