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Managerial turnover and entrenchment

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Abstract

We consider a two-period model in which the success of the firm depends on the effort of a first-period manager (the incumbent) as well as the effort and ability of a second-period manager. At the end of the first period, the board receives a noisy signal of the incumbent manager's ability and decides whether to retain or replace the incumbent manager. We show that severance pay can be utilized in the optimal contract to provide a credible commitment to a lenient second-period equilibrium replacement policy, mitigating the first-period moral hazard problem. Unlike existing models that aim to rationalize managerial entrenchment, we identify conditions on the information structure under which both entrenchment and anti-entrenchment emerge in the optimal contract. Specifically, our model predicts that it is optimal for the board to design a contract to induce entrenchment (respectively, anti-entrenchment) if the signal regarding the incumbent manager's ability becomes sufficiently uninformative (respectively, informative).

1 | INTRODUCTION

Designing compensation schemes in managerial contracts and deciding whether to replace a manager, such as a CEO, are important firm organization activities. These decisions are linked through the severance agreement, a key component of the contracts between boards and managers. The severance agreement specifies payments to the manager upon his forced departure. Approximately, 50% of the CEO compensation contracts implemented between 1994 and 1999 involved some form of severance agreement (Rusticus, 2006). The percentage of S&P 500 firms that included a severance agreement in their CEO compensation contracts increased from 20% in 1993 to more than 55% in 2007 (Huang, 2011). In general, a contract with a severance agreement adds an explicit cost to the board's retention decision and makes replacement more difficult relative to a compensation contract without such an agreement.

A widely held belief is that CEOs are replaced too infrequently, that is, they become entrenched.[1] Entrenchment may arise for many reasons. For example, it may be due to governance failure in the form of a captive board of directors (Hermalin & Weisbach, 1998; Inderst & Mueller, 2010; Shleifer & Vishny, 1989) or a way to mitigate a moral hazard problem (Almazan & Suarez, 2003; Casamatta & Guembel, 2010; Manso, 2011). Taylor (2010) makes the first attempt to measure the cost of entrenchment using a structural model of CEO turnover and finds suggestive evidence of the opposite. In particular, he finds that boards in large firms fire CEOs with higher frequency than is optimal. We refer to this phenomenon as *anti-entrenchment*.

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This finding cannot be rationalized by the existing models on CEO turnover and thus calls for a new model to better understand the determinants of managerial turnover.

This paper investigates how optimal design of the severance agreement influences managerial entrenchment. A manager is said to be entrenched (or anti-entrenched) if the board retains (fires) him when his expected ability is lower (higher) than that of a replacement manager. We propose a two-period principal–agent model of managerial turnover and identify conditions that predict the emergence of entrenchment and anti-entrenchment. Formally, we consider a setup in which the first-period manager is incentivized by a contract that contains performance-related pay and severance pay. The firm's success depends on the initial manager's effort, the second-period manager's effort and his ability. Thus, the board faces an ability selection problem and a period-1 moral hazard problem. After the initial manager exerts effort, the board observes a *noncontractible* signal regarding his ability. The board can fire the initial manager by paying the severance pay specified in the contract and hire a replacement manager.

Severance pay can be utilized in the optimal contract to provide a credible commitment to a lenient period-2 equilibrium replacement policy, which mitigates the period-1 moral hazard problem and influences the period-2 manager's expected ability. To see how the use of severance pay can be desirable for the board, it is useful to consider a contract with zero severance pay, under which the board is able to dismiss the incumbent at will. Because the wage is only paid at the end of the second period, the board will always prefer to fire the first-period manager if he is of the same ability as the replacement manager, so as to avoid paying the higher promised wage. Such a zero-severance-pay contract, however, may not be in line with the interest of the board. With a contract of zero severance pay in hand, the incumbent manager would expect an aggressive replacement policy and thus would not exert much first-period effort. In such a case, the board can commit to a positive severance payment, ensuring a low expected profit for itself after replacement. This would in turn lead to a less aggressive replacement policy, provide more job security to the incumbent manager, and thus mitigate the period-1 moral hazard problem at the cost of the expenditure from the severance agreement. Meanwhile, the introduction of a positive severance pay in the contract affects the expected ability of the second-period manager: the higher the severance pay, the lower the expected ability of the manager who is retained to save the cost. Therefore, the optimal contract must strike a balance between incentive provision, manager ability selection, and commitment cost.

Our main result characterizes the optimal replacement policy and shows how it depends on the precision of the signal of the manager's ability. When the monitoring technology is noisy, entrenchment is optimal. In such a scenario, the board places higher priority on motivating the incumbent manager to exert period-1 effort than on maximizing the manager's ability. Designing a contract to induce an aggressive replacement policy will too often result in the firing of the incumbent of high ability and will disincentivize the incumbent to exert effort, while saving little on severance pay. As a result, a contract that induces entrenchment is optimal for the board.

Anti-Entrenchment is optimal when the board's monitoring technology is sufficiently informative. On the one hand, because the board's monitoring technology is informative, the incumbent manager expects that the turnover rate relies mainly on his ability rather than the replacement policy. Therefore, a less aggressive replacement policy does not have a significant impact on motivating period-1 effort. On the other hand, through an aggressive replacement policy the board can (i) avoid the cost of severance pay when replacement occurs; and (ii) avoid paying the wage to the incumbent manager and undercut the wage in period 2 to further increase the firm's profit after the incumbent manager exerts his period-1 effort. Thus, anti-entrenchment is optimal for the board. To the best of our knowledge, we are the first to study the interaction between the board's monitoring technology and managerial turnover, and to show that anti-entrenchment can be part of the optimal contract.

1.1 | Related literature

This paper belongs to the literature on the principal–agent model with replacement.[2] One strand of research views entrenchment as a potential source of inefficiency that the board aims to mitigate. Consequently, anti-entrenchment cannot be observed. Inderst and Mueller (2010) solve the optimal contract for the incumbent manager who holds private information on the firm's future performance and can avoid replacement by concealing bad information. Consequently, the optimal contract is designed to induce the incumbent to voluntarily step down when evidence suggests low expected profit under his management. Similarly, entrenchment occurs if the incumbent can make manager-specific investments that create replacement costs for the board (Shleifer & Vishny, 1989) or if there exist close ties between the board and manager (Hermalin & Weisbach, 1998).

Another strand of research views entrenchment as a feature of the optimal contract (board structure) that helps overcome the moral hazard problem. Manso (2011) shows that tolerance for early failure (entrenchment) can be part of the optimal incentive scheme when motivating a manager to pursue more innovative business strategies is important to the board. Casamatta and Guembel (2010) study the optimal contract for the incumbent manager who is concerned about his reputation. In their model, entrenchment is optimal because the incumbent manager would like to see his strategy succeed and thus he is less costly to motivate than the replacement manager. Almazan and Suarez (2003) use a two-period model to study the optimal board structure for incentivizing the incumbent manager. In their model, an incumbent manager can exert effort to improve the effectiveness of his management in period 1, and a potential *better* replacement may arrive in period 2. The board can choose between the following two governance structures: a weak board where the incumbent can veto his departure, and a strong board where the board can fire the incumbent at will. Their model is similar to ours in that severance pay provides credible job security to the incumbent and can in turn motivate period-1 effort under each governance structure. Thus the board may be entrenched in equilibrium, that is, the incumbent is retained if the replacement manager is modestly better than him. It is useful to point out that the model of Almazan and Suarez (2003) rules out the possibility of anti-entrenchment because the replacement manager is always assumed to perform better than the incumbent even if the incumbent exerts effort in period 1.[3] In the same spirit, Laux (2008) studies the optimal degree of board independence for shareholders and shows that some lack of independence can increase shareholder value. In these papers, boards (shareholders) provide better job security to the incumbent by making dismissal more difficult in order to induce more effort. Our paper contributes to the existing literature by pointing out that despite all the incentive-providing merits of entrenchment, the cost of incentivizing can be high when the board's monitoring technology is sufficiently informative.

The main economic mechanism of this paper relates to the literature that investigates the tension between *ex post* optimality and *ex ante* incentive within an agency setting. Some papers demonstrate the optimality of entrenchment based on this trade-off (e.g., Almazan & Suarez, 2003; Burkart, Gromb, & Panunzi, 1997, among others). Other papers show that the *ex post* optimal decision may be too soft from an *ex ante* perspective (e.g., Crémer, 1995, among others), so that implementing the *ex ante* optimal policy can be interpreted as generating anti-entrenchment, as in this paper.

The main trade-off between moral hazard and selection at the core of this paper is reminiscent of that in Inderst and Klein (2007) concerning corporate budgeting. They study a model in which a division manager exerts effort to generate a new project, and is privately informed about its prospects before the firm invests capital to realize this opportunity. Because the investment is costly to the firm, the division manager sets a lower standard on project selection than the firm would choose. In other words, the manager has a tendency to overinvest.[4] Similarly, in this paper, after the incumbent manager exerts effort, the board has a propensity to excessively replace the incumbent due to the promised performance-related pay. Unlike Inderst and Klein (2007), because the contract in our framework is endogenously designed by the board, both entrenchment and anti-entrenchment can emerge.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the first best outcome and defines entrenchment and anti-entrenchment accordingly. Section 4 analyzes the optimal contract in a simplified model in which the period-2 moral hazard problem is absent, and studies the consequences of informativeness on the equilibrium replacement policy under the optimal contract. Section 5 investigates the full model and shows that the results we derived in Section 4 are robust. Section 6 discusses some of our modeling assumptions and presents some extensions of the model. Section 7 summarizes our main findings and concludes. All proofs are presented in the Appendix.

2 | THE MODEL

There are two periods $t = 1, 2$ and an initial contract stage ($t = 0$).

Contract stage

The board (principal), hires a manager (agent) from a pool with unknown ability $\theta_1 \in \{0, 1\}$ to work for the firm with common prior $\Pr(\theta_1 = 1) = \frac{1}{2}$.[5] The analysis is unchanged for a different prior of θ_1. The manager's ability is unknown to both sides. The board offers a contract to the manager. We describe the contract details below.

Both the board and the managers are risk-neutral. Moreover, we assume that managers are protected by limited liability.[6] Finally, we assume the value of the outside option to the manager is 0. This assumption guarantees that the individual rationality (IR) constraint never binds and simplifies the analysis.

Period 1

The incumbent manager exerts effort $e_1 \in [0, 1]$ in the first period. Simultaneously, the board receives a signal $s \in \mathcal{S}$ of the manager's ability and decides whether to replace the incumbent manager. If the incumbent manager is fired, a replacement manager is hired and has ability θ_r randomly drawn from the same pool of managers. In the rest of the paper, we use variables with subscript r to indicate "replacement manager."

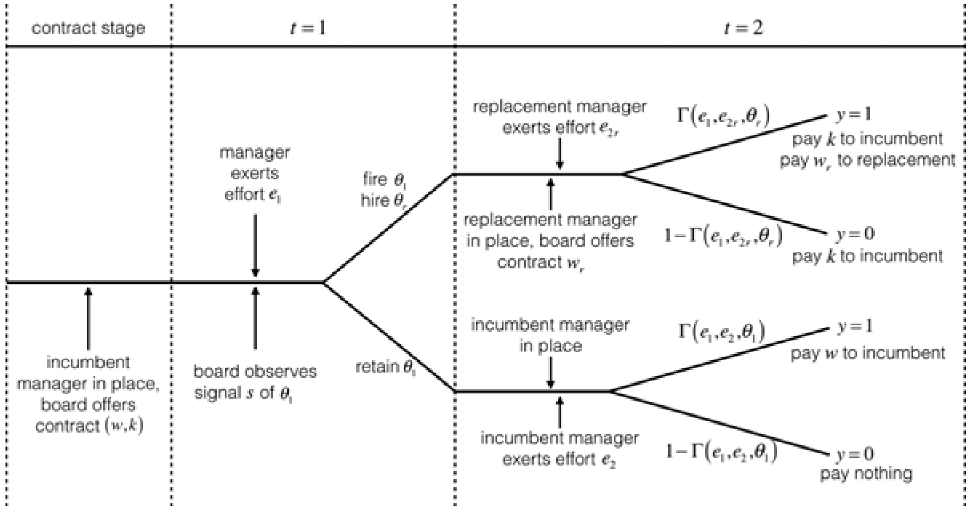


FIGURE 1 Timeline

Period 2

If the incumbent manager is retained, he exerts effort $e_2 \in [0, 1]$. Similarly, if replacement occurs, the board offers a new contract to the replacement manager and the replacement manager exerts effort $e_{2r} \in [0, 1]$. The cost function to the incumbent manager is assumed to be $C_i(e_1, e_2) := \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$, and the cost function to the replacement manager is assumed to be $C_r(e_{2r}) := C_i(0, e_{2r}) = \frac{1}{2}e_{2r}^2$. The quadratic cost function is analytically helpful but not crucial to the qualitative nature of the main results. The outcome depends on the period-1 effort, the period-2 effort, and the ability of the period-2 manager. Specifically, we assume the success ($y = 1$) probability is equal to

$$\Gamma(e_1, \tilde{e}_2, \tilde{\theta}_2) := \tilde{\theta}_2 \left[(1 - \lambda)e_1 + \lambda\tilde{e}_2\right], \tag{1}$$ [7,8]

where $\tilde{e}_2 \in [0, 1]$ and $\tilde{\theta}_2 \in \{0, 1\}$ are the effort and ability of the manager who remains in office in period 2, respectively, and $\lambda \in [0, 1]$ measures the relative importance of the period-2 effort. Specifically, $(\tilde{e}_2, \tilde{\theta}_2) = (e_{2r}, \theta_r)$ if the incumbent manager is replaced and $(\tilde{e}_2, \tilde{\theta}_2) = (e_2, \theta_1)$ otherwise. With probability $\Gamma(e_1, \tilde{e}_2, \tilde{\theta}_2)$, the project is of high quality and yields outcome $y = 1$. With complementary probability $1 - \Gamma(e_1, \tilde{e}_2, \tilde{\theta}_2)$, the project is of low quality and yields outcome $y = 0$. After payoffs are realized, the incumbent manager and the replacement manager (if replacement occurs) receive payment according to the contracts signed in period 0 and period 2, respectively, and the game comes to an end. The timeline of the events is summarized in Figure 1.

It is useful to point out that the complementarity between ability and effort assumed in expression (1) is not crucial for our main results on entrenchment and anti-entrenchment derived in Propositions 1 and 2. In fact, this assumption allows us to investigate the interesting interactions between information about the incumbent's ability and his incentive to exert effort in period 2. If ability and effort are substitutes, then the incumbent manager's period-2 effort will not depend on the signal. As a result, the learning effect and the sorting effect discussed later in Section 5 will be absent and the main results remain qualitatively unchanged.

Contract

It is straightforward to see that in the optimal contract, the wage for low output to both the incumbent manager and the potential replacement manager is equal to 0. Therefore, a contract to the incumbent manager is defined by the tuple (w, k), where w is the wage rate when $y = 1$ and k is the lump-sum severance pay to the incumbent manager if he is fired.[9] By the limited liability assumption, $w \geq 0$ and $k \geq 0$. Similarly, a contract to the replacement manager is indexed by $w_r \geq 0$, where w_r is the wage rate when $y = 1$.

Information structure

The board receives a noisy signal $s \in \mathcal{S}$ regarding the incumbent manager's ability θ_1 in the first period, which is drawn from a distribution with cumulative distribution function (CDF) $F_{\theta_1}(\cdot)$ and probability density function (PDF) $f_{\theta_1}(\cdot)$ for $\theta_1 \in \{0, 1\}$. Without loss of generality, we assume $\mathcal{S} = [0, 1]$. The two conditional density functions $\{f_1(s), f_0(s)\}$ suffice to define an information structure under such normalization. Three assumptions are imposed on the information structure.

Assumption 1. The monotone likelihood ratio property (MLRP): $\frac{f_1(s)}{f_0(s)}$ is strictly increasing in s for $s \in [0, 1]$.

For binary states, the MLRP assumption is without loss of generality because signals can always be relabeled according to likelihood ratio to satisfy this assumption.

Assumption 2. Perfectly informative at extreme signals: $\lim_{s\to 0} \frac{f_1(s)}{f_0(s)} = 0$ and $\lim_{s\to 1} \frac{f_1(s)}{f_0(s)} = +\infty$.

Assumption 2 guarantees that support of the posterior belief is always [0,1]. The last assumption imposed on the information structure is symmetry. This assumption allows us to define the first best replacement policy on the signal space.

Assumption 3. $f_1(s) = f_0(1 - s)$ for all $s \in [0, 1]$.

It follows directly from Assumption 3 that $f_1(\frac{1}{2}) = f_0(\frac{1}{2})$. Thus, the likelihood ratio at $s = \frac{1}{2}$ is always 1 and the board's estimate of the incumbent manager's ability at $s = \frac{1}{2}$ is equal to the prior.

Finally, we introduce an index $\alpha \in (0, \infty)$ to parameterize the information structure. We assume that $f_{\theta_1}(s; \alpha)$ is continuous in s and α for $\theta_1 \in \{0, 1\}$ and define the information structures for the two extreme values of α as follows.

Assumption 4 (Extreme Information Structures).

(i) The information structure becomes completely uninformative as $\alpha \to 0$, that is, $\lim_{\alpha\to 0}[f_0(s; \alpha) - f_1(s; \alpha)] = 0$ for all $s \in (0, 1)$.

(ii) The information structure becomes completely informative as $\alpha \to \infty$, that is, $\lim_{\alpha\to\infty} f_1(s; \alpha) = 0$ for all $s \in [0, \frac{1}{2})$ and $\lim_{\alpha\to\infty} f_0(s; \alpha) = 0$ for all $s \in (\frac{1}{2}, 1]$.[10]

In words, when the information structure becomes completely uninformative ($\alpha \to 0$), the two conditional density functions are the same. When the information structure becomes completely informative ($\alpha \to \infty$), the board does not observe a signal below $\frac{1}{2}$ if the incumbent manager is of high ability and a signal above $\frac{1}{2}$ if the incumbent manager's ability is low.

Two remarks are in order before we proceed to the analysis. First, following Holmström (1999), we assume that the incumbent manager does not have any private information about his ability when he signs the contract, an assumption which is widely adopted in the literature (see also Bonatti & Hörner, 2017; Fang & Moscarini, 2005, among others). One way of justifying this assumption is that the manager's ability in our model is firm-specific and can be interpreted as the match quality between the board and the manager. Second, to emphasize the effect of signal noncontractibility, we assume in the baseline model that the signal is learned by both parties but is not contractible by the board at the beginning of the second stage. In Section 6.2, we show that our results continue to hold when the signal is the board's private information and cannot be observed by the incumbent manager.

3 | THE BENCHMARK CASE: CONTRACTIBLE EFFORT

We first pin down the socially optimal replacement policy, and define entrenchment and anti-entrenchment accordingly. We define the first-best outcome as the outcome in the absence of the moral hazard problem, but with an information problem. That is, the board only has the noisy signal s on which to base its decision to fire or retain the manager, and how much effort to specify in period 2. It follows from Assumption 1 that the socially optimal replacement policy is a cutoff rule. Denote $\hat{s}$ as the signal cutoff. The board chooses $(e_1, e_2(s), e_{2r}, \hat{s})$ to maximize

$$\max_{\{e_1, e_2(s), e_{2r}, \hat{s}\}} \int_{\hat{s}}^{1} \left\{ \frac{f_1(s)}{f_1(s) + f_0(s)} \left[(1-\lambda)e_1 + \lambda e_2(s)\right] - \frac{1}{2}[e_2(s)]^2 \right\} d\frac{F_1(s) + F_0(s)}{2}$$
$$+ \frac{1}{2}\left[F_1(\hat{s}) + F_0(\hat{s})\right] \times \left[\frac{(1-\lambda)e_1 + \lambda e_{2r}}{2} - \frac{1}{2}e_{2r}^2\right] - \frac{1}{2}e_1^2.$$

Lemma 1 (First-Best Outcome). *Suppose that the board can contract on the effort $(e_1, e_2(s))$ of the incumbent manager and e_{2r} of the replacement manager. Then, the optimal period-1 effort is $e_1^{\mathcal{FB}} = \frac{1}{2}(1-\lambda)\{1 + \frac{1}{2}[F_0(\hat{s}^{\mathcal{FB}}) - F_1(\hat{s}^{\mathcal{FB}})]\}$. In period 2, the incumbent is retained and exerts effort $e_2^{\mathcal{FB}}(s) = \frac{f_1(s)}{f_1(s)+f_0(s)}\lambda$ if $s > \hat{s}^{\mathcal{FB}}$. Otherwise, the board hires the replacement and enforces $e_{2r}^{\mathcal{FB}} = \frac{1}{2}\lambda$. Moreover, the optimal replacement cutoff is $\hat{s}^{\mathcal{FB}} = \frac{1}{2}$.*

When effort is contractible, the board is able to optimize effort and ability selection separately. Thus, there is no trade-off between the moral hazard problem and the ability selection problem. It is optimal to replace the incumbent manager when the posterior belief about the incumbent's ability falls below the expected value of the pool, otherwise it is optimal to retain the incumbent. By Assumption 3, the likelihood ratio $\frac{f_1(s)}{f_0(s)}$ at the neutral signal $s = \frac{1}{2}$ is always equal to 1, which in turn implies that the Bayesian update of the incumbent manager's ability is always equal to the prior independent of the informativeness α of the information structure. Consequently, the socially optimal cutoff $\hat{s}^{\mathcal{FB}} = \frac{1}{2}$.

Denote (w^*, k^*) and w_r^* as the period-1 and period-2 optimal contracts to the board, respectively. Let $(\hat{s}^*, e_1^*, e_2^*(s), e_{2r}^*)$ be the equilibrium replacement cutoff and effort in the continuation game induced by the optimal contract. Given the first-best cutoff derived in Lemma 1, we can now define entrenchment and anti-entrenchment as follows:

Definition 1. We define "entrenchment" as a cutoff $\hat{s}^* < \frac{1}{2}$ and "anti-entrenchment" as $\hat{s}^* > \frac{1}{2}$.

For the case where $\hat{s}^* = \frac{1}{2}$, we say that neither entrenchment nor anti-entrenchment is observed. In this case, the replacement policy coincides with the socially optimal policy. When $\hat{s}^* < \frac{1}{2}$, the replacement policy favors the incumbent manager: the board could have improved the expected ability of the period-2 manager by replacing the incumbent. Similarly, the replacement policy is considered aggressive and places the incumbent manager at a disadvantage when $\hat{s}^* > \frac{1}{2}$.

4 | OPTIMAL CONTRACT AND REPLACEMENT POLICY: $\lambda = 0$

In this section, we solve the equilibrium outcome when effort is noncontractible. That is, the board can only commit to the wage w and severance pay k in the contract. To explain the intuition most cleanly, let us first consider the case where $\lambda = 0$. In Section 5, we allow λ to be positive and again show that the main results derived in this section continue to hold.

When $\lambda = 0$, the success probability in expression (1) degenerates to $\tilde{\theta}_2 e_1$ and does not depend on the period-2 effort $\tilde{e}_2$. Therefore, the board will not offer a contract to the replacement manager, or equivalently, $w_r^* = 0$ in the optimal contract. Next, we solve the game by backward induction. We are interested in the cutoff $\hat{s}^*$ induced by the optimal contract.

Incentives under fixed contract (w, k)

A contract (w, k) induces a sub-game in which the manager chooses effort and the board chooses the replacement cutoff simultaneously. The incumbent manager's effort e_1 and the board's replacement policy $\hat{s}$ will be determined in a Cournot–Nash equilibrium.

For contract (w, k), the incumbent manager's best response to cutoff $\hat{s}$ is effort e_1 that maximizes

$$\max_{e_1} \frac{1}{2}\left[1 - F_1(\hat{s})\right] e_1 w + \frac{1}{2}\left[F_1(\hat{s}) + F_0(\hat{s})\right] k - \frac{1}{2}e_1^2.$$

With probability $\frac{1}{2}[1 - F_1(\hat{s})]$, the incumbent manager is of high ability ($\theta_1 = 1$) and is retained; and his expected payoff from exerting effort e_1 is $\tilde{\theta}_2 e_1 w = e_1 w$.[11] With probability $\frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]$, the incumbent is replaced and receives a lump-sum severance pay of k. The first-order condition for the above maximization problem with respect to e_1 yields

$$e_1(\hat{s}; w, k) = \frac{1}{2}\left[1 - F_1(\hat{s})\right] w.\text{[12]} \tag{2}$$

From equation (2), the board can provide incentive to exert effort by increasing wage w directly or by designing a contract that indirectly induces a lower equilibrium cutoff $\hat{s}$.

Similarly, for a fixed contract (w, k), the board's best response to the incumbent manager's effort level e_1 is to choose a cutoff $\hat{s}$ to maximize the expected profit

$$\max_{\hat{s}} \frac{1}{2}\left[1 - F_1(\hat{s})\right] e_1(1 - w) + \frac{1}{2}\left[F_1(\hat{s}) + F_0(\hat{s})\right]\left(\frac{1}{2}e_1 - k\right).$$

With probability $\frac{1}{2}[1 - F_1(\hat{s})]$, the incumbent manager is of high ability ($\theta_1 = 1$) and is retained, and the board pays w when $y = 1$. Therefore, the corresponding expected profit is $\tilde{\theta}_2 e_1(1 - w) = e_1(1 - w)$. With probability $\frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]$, a replacement manager with expected ability $\mathbb{E}(\theta_r) = \frac{1}{2}$ is in office. In this case, the board's expected profit is the difference between the expected outcome, which is $\mathbb{E}(\theta_r)e_1 = \frac{1}{2}e_1$, and the severance pay k. The board's replacement cutoff, denoted by $\hat{s}(e_1; w, k)$, can be derived from the following indifference condition:[13]

$$\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} e_1(1 - w) = \frac{1}{2}e_1 - k. \tag{3}$$

The left-hand side of equation (3) is the expected profit when the board retains the incumbent manager after observing signal $\hat{s}$; and the right-hand side is the board's expected profit upon replacement. Because a higher cutoff implies higher posterior belief about the incumbent manager's ability, the board chooses a cutoff such that the expected profit created by the marginal incumbent manager is equal to the expected profit when replacement occurs in equilibrium.

To better understand the board's period-2 equilibrium replacement decision and its period-1 incentive in designing the contract (w, k), it is useful to rewrite equation (3) as follows:

$$\underbrace{\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} e_1}_{\substack{\text{Expected revenue of retaining} \\ \text{the incumbent manager}}} - \underbrace{\frac{1}{2}e_1}_{\substack{\text{Expected revenue of replacing} \\ \text{the incumbent manager}}} = \underbrace{\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} e_1 w}_{\substack{\text{Expected cost of retaining} \\ \text{the incumbent manager}}} - \underbrace{k}_{\substack{\text{Expected cost of replacing} \\ \text{the incumbent manager}}}.$$

Note that although the replacement manager receives no payment in the case where $\lambda = 0$, replacing the incumbent manager incurs a cost of k to the board. Now suppose that the board receives a neutral signal $s = \frac{1}{2}$ and thus $\frac{f_1(s)}{f_1(s)+f_2(s)} = \frac{1}{2}$. Then the incumbent manager and the potential replacement manager are of the same expected ability and generate the same expected revenue (i.e., $\frac{f_1(s)}{f_1(s)+f_0(s)}e_1 = \frac{1}{2}e_1$) to the board. Under such a scenario, whether entrenchment or anti-entrenchment emerges in equilibrium depends solely on the comparison of the expected costs from the above condition. Specifically, if $\frac{1}{2}e_1 w > k$, then replacing the incumbent manager with a neutral signal is optimal because the board can (i) avoid paying the wage w by the end of period 2 through replacement; and (ii) instead pay the incumbent manager a low severance pay k (and pay nothing to the replacement in the case where $\lambda = 0$[14]). As a result, anti-entrenchment arises in equilibrium.[15] Similarly, retaining the incumbent manager is preferred by the board if $\frac{1}{2}e_1 w < k$ and entrenchment arises. Note that both w and k are endogenous variables and are specified by the board in its period-1 contract to the incumbent manager, and hence whether entrenchment or anti-entrenchment arises in equilibrium under the optimal contract is nontrivial.

Given contract (w, k), the equilibrium cutoff and effort $(\hat{s}(w, k), e_1(w, k))$ are pinned down by equations (2) and (3). Alternatively, we can calculate the corresponding contract (w, k) that induces any tuple $(\hat{s}, e_1)$ as follows:

$$w(\hat{s}, e_1) = \frac{e_1}{\frac{1}{2}\left[1 - F_1(\hat{s})\right]}, \tag{4}$$

and

$$k(\hat{s}, e_1) = \frac{1}{2}e_1 - \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} e_1 \left[1 - w(\hat{s}, e_1)\right]. \tag{5}$$

It follows immediately from equation (4) that the limited liability constraint for w is satisfied. However, the limited liability constraint for k is not always satisfied for all $(\hat{s}, e_1) \in [0, 1] \times [0, 1]$ from equation (5). Next, we proceed by solving for a relaxed problem where the limited liability constraint on k is dropped. It can be verified later that $k \geq 0$ is satisfied (see Lemma 2) at the optimal contract in the relaxed problem, implying that the optimal contract in the relaxed problem also solves the original problem where the limited liability constraint is imposed.

Analysis of the first stage game

The board chooses contract (w, k) to maximize expected profit as follows:

$$\max_{\{w,k\}} \frac{1}{2}[1 - F_1(\hat{s})]e_1(1 - w) + \frac{1}{2}\left[F_1(\hat{s}) + F_0(\hat{s})\right]\left(\frac{1}{2}e_1 - k\right)$$

s.t.

$$e_1 = \frac{1}{2}\left[1 - F_1(\hat{s})\right] w,$$

and

$$\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}e_1(1-w)=\frac{1}{2}e_1-k.$$

Equivalently, the board is maximizing expected profit over the equilibrium variables $(\hat{s}, e_1)$, with $w(\hat{s}, e_1)$ and $k(\hat{s}, e_1)$ as determined in equations (4) and (5). Substituting equations (4) and (5) into the board's profit function yields the expected profit as a function of $(\hat{s}, e_1)$

$$e_1\left[1-\frac{e_1}{\frac{1}{2}\left[1-F_1(\hat{s})\right]}\right]\left\{\frac{1}{2}\left[1-F_1(\hat{s})\right]+\frac{1}{2}\left[F_1(\hat{s})+F_0(\hat{s})\right]\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}\right\}.$$

It can be verified that $e_1=\frac{1}{4}[1-F_1(\hat{s})]$ under the optimal contract. Consequently, $w^*=\frac{1}{2}$. The following lemma summarizes the above discussions.

Lemma 2. *Fixing a replacement cutoff $\hat{s}$ that it would like to induce, the board maximizes expected profit by offering a contract $w=\frac{1}{2}$ and $k(\hat{s})=\frac{1}{4}[1-F_1(\hat{s})][\frac{1}{2}-\frac{1}{2}\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}]\geq 0$. Moreover, in equilibrium, the incumbent manager chooses effort $e_1(\hat{s})=\frac{1}{4}[1-F_1(\hat{s})]$.*

Note that $k(\hat{s})$ is strictly decreasing in the equilibrium cutoff $\hat{s}$ from Lemma 2. Therefore, by committing to a higher severance pay, the board chooses a lower replacement cutoff in equilibrium and hence is able to induce greater effort [see equation (2)]. Exploiting the results in Lemma 2, the expected profit can be rewritten in terms of $\hat{s}$ alone:

$$\pi(\hat{s}):=\frac{1}{8}\underbrace{\left[1-F_1(\hat{s})\right]}_{\text{incentive effect}}\times\left\{\underbrace{\left[\frac{1}{2}[1-F_1(\hat{s})]+\frac{1}{4}[F_1(\hat{s})+F_0(\hat{s})]\right]}_{\text{selection effect}}\right.$$
$$\left.+\underbrace{\frac{1}{2}[F_1(\hat{s})+F_0(\hat{s})]\left(\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}-\frac{1}{2}\right)}_{\text{commitment cost effect}}\right\}.$$

It is clear that the optimal cutoff relies on the informativeness of the information structure. From the above expression, three effects play a role in determining the optimal cutoff. Because the outcome depends on the expected ability of the manager in period 2, the board faces an ability selection problem. This is captured by the term $[\frac{1}{2}[1-F_1(\hat{s})]+\frac{1}{4}[F_1(\hat{s})+F_0(\hat{s})]]$, which is called the *selection effect*. In fact, this term is exactly the expected ability of the manager that remains in office in period 2.[16] To maximize this term, the board would replace the incumbent if the posterior belief of his ability falls below the prior, or equivalently, if the signal is below $\frac{1}{2}$, and would retain the incumbent otherwise. Therefore, to independently optimize ability selection, the board would select a contract to induce $\hat{s}=\frac{1}{2}$.

Because the outcome also depends on the effort choice of the incumbent manager, the board faces a moral hazard problem and needs to incentivize the incumbent. This is captured by the term $[1-F_1(\hat{s})]$, which is referred to as the *incentive effect*. As the equilibrium replacement cutoff $\hat{s}$ increases, the incumbent manager expects a lower probability of retention in equilibrium and exerts less effort accordingly [see equation (2)]. In response, the board provides greater job security in order to better incentivize the incumbent manager. By this effect alone, the board induces $\hat{s}=0$.

If the selection effect and the incentive effect were the only effects, a cutoff below $\frac{1}{2}$ would be optimal to the board and entrenchment would emerge under the optimal contract. However, because the signal is noncontractible, the board lacks commitment power on the replacement policy. Severance pay serves as a costly commitment device that helps make replacement of the incumbent less likely. As the severance pay increases, it lowers the expected payoff of replacement, which creates a stronger incentive for the board to retain the incumbent. In equilibrium, the expected profit of replacement is equal to the expected profit created by the marginal incumbent manager. When the board lowers the cutoff ($\hat{s}<\frac{1}{2}$) to provide greater incentive on effort, it has to increase severance pay to make the equilibrium replacement policy credible. This generates a net loss compared to the first-best replacement policy due to the increment of the severance pay. It is captured by the term $\frac{1}{2}[F_1(\hat{s})+F_0(\hat{s})](\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}-\frac{1}{2})$, which is referred to as the *commitment cost effect*. Compared to the first best cutoff $\hat{s} = \frac{1}{2}$, the board saves severance pay by providing less commitment and designing a contract that induces a cutoff above $\frac{1}{2}$. By the same token, the board bears a greater severance pay cost by committing to a cutoff that is below $\frac{1}{2}$. The net commitment cost effect is shown by the term $(\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})} - \frac{1}{2})$. Multiplied by the probability of replacement, this yields the total net commitment gain/loss. By this effect alone, the board induces $\hat{s} = 1$.

To summarize, whether entrenchment or anti-entrenchment is optimal to the board depends on the strength of the incentive effect and the commitment cost effect. If the incentive effect dominates the commitment cost effect, entrenchment is optimal. Otherwise, anti-entrenchment is optimal. The next proposition illustrates how the optimal replacement policy varies depending on the informativeness of the board's monitoring technology.

Proposition 1. *Suppose that $\{f_1(\cdot;\alpha), f_0(\cdot;\alpha)\}$ satisfies Assumptions 1–4. Then there exist two thresholds $\overline{\alpha}$ and $\underline{\alpha}$ such that: (i) $\hat{s}^*(\alpha) > \frac{1}{2}$ for $\alpha > \overline{\alpha}$; (ii) $\hat{s}^*(\alpha) < \frac{1}{2}$ for $\alpha < \underline{\alpha}$.*[17]

When the information structure is noisy, providing incentive is more profitable than avoiding the severance pay through providing less commitment on the equilibrium replacement policy, and thus entrenchment arises under the optimal contract. To see this more clearly, suppose that the board offers the incumbent manager a contract that induces an equilibrium cutoff signal at $\frac{1}{2}$, and is considering increasing the cutoff signal locally by a small amount $\epsilon > 0$. By the incentive effect, increasing the equilibrium cutoff signal to $\hat{s} = \frac{1}{2} + \epsilon$ reduces the incumbent manager's incentive to exert period-1 effort from (2) and the board's profit decreases. On the other hand, increasing the equilibrium cutoff helps the board avoid the severance pay and leads to an increase in profits by the commitment cost effect. Recall that the amount of savings on the severance pay (i.e., the strength of the commitment cost effect) depends on the size of $(\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})} - \frac{1}{2})$, which in turn depends on the informativeness of the information structure. For a noisy information structure (i.e., $\alpha \to 0$), the expected ability of the incumbent manager at the cutoff signal $\hat{s} = \frac{1}{2} + \epsilon$ is very close to that at $\frac{1}{2}$, implying that the amount of severance pay that can be saved by locally increasing the equilibrium cutoff is negligible. Therefore, the incentive effect dominates the commitment cost effect and thus it is optimal for the board to design a contract that leads to entrenchment in equilibrium.

When the board's monitoring technology becomes sufficiently informative, the commitment cost effect takes over and inducing an equilibrium cutoff signal below $\frac{1}{2}$ is not optimal for the board. Similar to the analysis for the case where $\alpha \to 0$, suppose that the board is offering a contract to the incumbent manager that induces an equilibrium cutoff signal at $\frac{1}{2}$, and is considering decreasing the cutoff signal locally by a small amount $\epsilon > 0$. Because the probability of firing a high-ability manager is very small for all signals below $\frac{1}{2}$ from Assumption 4, reducing the equilibrium replacement cutoff to less than $\frac{1}{2}$ has little effect on the incumbent's period-1 effort. Furthermore, decreasing the equilibrium cutoff signal requires the board to increase the severance pay significantly. To see this, note that the expected ability of the manager in the left neighborhood of $\frac{1}{2}$ is very close to 0 as $\alpha \to 0$. In other words, the board is very certain that the incumbent manager is of low ability when it receives a signal slightly below $\frac{1}{2}$. In order for the board to be indifferent between replacing and retaining the incumbent upon receiving that signal, a significantly higher severance pay has to be promised relative to the case where the board would like to induce an equilibrium cutoff signal equal to $\frac{1}{2}$. Therefore, the board has no incentive to design a contract to reduce the equilibrium cutoff below $\frac{1}{2}$, and anti-entrenchment is expected under the optimal contract.

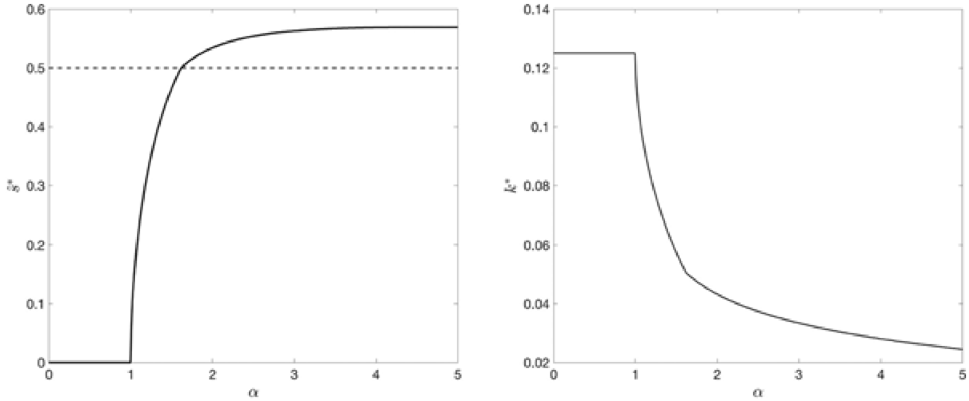
It is useful to point out that Assumption 4 defines informativeness when α becomes sufficiently large or small, and hence Proposition 1 is a limiting result. In Online Appendix A, we introduce an information order that we refer to as the ρ-concave order and we characterize the equilibrium replacement policy for all α.[18] We show that if the information structures can be ranked by the ρ-concave order, then there exists a threshold of α above (respectively, below) which anti-entrenchment (respectively, entrenchment) emerges in equilibrium under the optimal contract. Next, we provide an example to illustrate this point.

Example 1 (A Family of Information Structures Ranked by ρ-Concavity Order). Suppose that $\lambda = 0$ and the board's signal, s, is drawn from one of the two densities on [0,1]:

$$f_1(s;\alpha) = \begin{cases} (2s)^\alpha & \text{for } s \in [0, \frac{1}{2}] \\ 2 - [2(1-s)]^\alpha & \text{for } s \in (\frac{1}{2}, 1], \end{cases}$$

and

$$f_0(s;\alpha) = \begin{cases} 2 - (2s)^\alpha & \text{for } s \in [0, \frac{1}{2}] \\ [2(1-s)]^\alpha & \text{for } s \in (\frac{1}{2}, 1]. \end{cases}$$



(a) Equilibrium Turnover under Optimal Contract. (b) Severance Pay under Optimal Contract.

FIGURE 2 Equilibrium turnover and severance pay under optimal contract

It can be verified that Assumptions 1–4 are satisfied. Moreover, the equilibrium replacement policy $\hat{s}^*(\alpha)$ is characterized as follows:

(i) if $\alpha \leq 1$, then $\hat{s}^*(\alpha) = 0$;

(ii) if $1 < \alpha < \frac{\sqrt{5}+1}{2}$, then $\hat{s}^*(\alpha) \in (0, \frac{1}{2})$;

(iii) if $\alpha > \frac{\sqrt{5}+1}{2}$, then $\hat{s}^*(\alpha) \in (\frac{1}{2}, 1)$.

Note that fixing α and the equilibrium replacement policy $\hat{s}^*(\alpha)$, the corresponding equilibrium turnover is $\frac{1}{2}[F_1(\hat{s}^*(\alpha); \alpha) + F_0(\hat{s}^*(\alpha); \alpha)]$. Exploiting the functional form of $\{f_1(\cdot; \alpha), f_0(\cdot; \alpha)\}$ in Example 1, it is straightforward to verify that the equilibrium turnover is equal to the equilibrium cutoff signal $\hat{s}^*(\alpha)$. Figure 2(a) depicts the equilibrium turnover for different informativeness levels of the monitoring technology. It is clear that turnover is increasing for $\alpha \in [1, \frac{\sqrt{5}+1}{2}]$ when the manager is entrenched. The relationship between turnover and the informativeness of the board's monitoring technology is an inverted-U shape for $\alpha > \frac{\sqrt{5}+1}{2}$. As α approaches infinity, the optimal cutoff converges to $\frac{1}{2}$. Figure 2(b) shows that severance pay in the optimal contract is decreasing in the informativeness of the board's monitoring technology. This result is intuitive: when the information structure becomes more informative, it is easier for the board to obtain the net commitment gain. Therefore, the board is less willing to commit to retaining the incumbent manager, and has incentive to reduce the amount of severance pay specified in the optimal contract.

5 | OPTIMAL CONTRACT AND REPLACEMENT POLICY: $\lambda > 0$

In this section, we allow λ to be positive. When $\lambda > 0$, the board needs to offer a new contract to the replacement manager if the incumbent manager is fired. As will be shown later, due to the fact that the incumbent manager needs to exert effort in period 2 in the event that he is retained, two new effects emerge and need to be taken into consideration when the board designs the contract. Again we can show that the main result derived in Proposition 1 continues to hold.

5.1 | Characterizing the optimal contract

Optimal contract with the replacement manager

We first calculate the optimal contract with the replacement manager w_r after the incumbent's departure for a given belief of period-1 effort e_1. It is useful to point out that (i) the replacement manager's effort decision e_{2r} is independent of the signal because the success function is independent of θ_1 after replacement; (ii) e_{2r} is independent of period-1 effort e_1 because efforts in the two periods are separable. Fixing w_r, the replacement manager chooses e_{2r} to maximize

$$\max_{e_{2r}} \frac{1}{2}\left[(1-\lambda)e_1 + \lambda e_{2r}\right] w_r - \frac{1}{2}e_{2r}^2 \Rightarrow e_{2r}(w_r) = \frac{1}{2}\lambda w_r. \quad (6)$$

Therefore, the board chooses w_r to maximize

$$\max_{w_r} \frac{1}{2}\left[(1-\lambda)e_1 + \lambda e_{2r}(w_r)\right](1-w_r) \Rightarrow w_r(e_1) = \max\left\{\frac{1}{2} - \frac{1-\lambda}{\lambda^2}e_1, 0\right\}.$$

Note that the optimal wage to the replacement manager is decreasing in the board's belief about e_1. When the period-1 effort e_1 is sufficiently large or λ is sufficiently small, the board provides a contract with $w_r = 0$ to the replacement manager. Let $\underline{\pi}(e_1)$ denote the board's expected profit excluding the severance pay with contract $w_r(e_1)$ after replacement. Simple algebra yields

$$\underline{\pi}(e_1) = \begin{cases} \frac{1}{4}\left(\frac{1}{2}\lambda + \frac{1-\lambda}{\lambda}e_1\right)^2 & \text{for } e_1 \le \frac{1}{2}\frac{\lambda^2}{1-\lambda} \\ \frac{1}{2}(1-\lambda)e_1 & \text{for } e_1 > \frac{1}{2}\frac{\lambda^2}{1-\lambda}. \end{cases} \tag{7}$$

Incentives under fixed contract (w, k)

A contract (w, k) induces a subgame in which the incumbent manager chooses period-1 effort, and period-2 effort $e_2(s)$ after observing signal s if he is retained, and the board chooses the replacement cutoff. Fixing a contract (w, k) and belief of $\hat{s}$, the incumbent manager chooses effort e_1 and $e_2(s)$ to maximize

$$\int_{\hat{s}}^{1}\left\{\frac{f_1(s)}{f_1(s)+f_0(s)}\left[(1-\lambda)e_1 + \lambda e_2(s)\right]w - \frac{1}{2}[e_2(s)]^2\right\} d\frac{F_1(s)+F_0(s)}{2} + \frac{F_1(\hat{s})+F_0(\hat{s})}{2}k - \frac{1}{2}e_1^2.$$

The first-order conditions for the above maximization problem with respect to e_1 and $e_2(s)$ yield

$$e_1(\hat{s}; w, k) = \frac{1-F_1(\hat{s})}{2}(1-\lambda)w, \tag{8}$$

and

$$e_2(s, \hat{s}; w, k) = \frac{f_1(s)}{f_1(s)+f_0(s)}\lambda w, \text{ if } s \ge \hat{s}. \tag{9}$$

As in the case where $\lambda = 0$, the board can motivate the period-1 effort by increasing wage w directly or by designing a contract to lower the equilibrium cutoff $\hat{s}$ indirectly from equation (8). Next we consider the period-2 effort. First, notice that upon observing the signal, the marginal manager with the cutoff signal $\hat{s}$ updates the belief about his ability from $\frac{1}{2}$ to $\frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}$ and exerts effort accordingly. This is referred to as the *learning effect* [see also equation (17)]. Second, because the signal is observable to the incumbent manager, he can condition his period-2 effort on the signal s. Moreover, the complementarity between effort and ability implies that $e_2(s, \hat{s}; w, k)$ is increasing in s [see equation (9)].[19] The additional amount of expected outcome motivated is

$$\begin{aligned} &\int_{\hat{s}}^{1}\left\{\left[e_2(s, \hat{s}; w, k) - e_2(\hat{s}, \hat{s}; w, k)\right] \times \frac{f_1(s)}{f_1(s)+f_0(s)}\right\} d\frac{F_1(s)+F_0(s)}{2} \\ &= \lambda w \int_{\hat{s}}^{1}\left[\left(\frac{f_1(s)}{f_1(s)+f_0(s)} - \frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}\right) \times \frac{f_1(s)}{f_1(s)+f_0(s)}\right] d\frac{F_1(s)+F_0(s)}{2}, \end{aligned} \tag{10}$$

which is referred to as the *sorting effect* [see also equation (17)]. In Section 5.2, we will discuss in detail how the magnitudes of these two new effects vary with the informativeness of the board's information structure.

For a fixed contract (w, k) and belief about the effort profile $(e_1, e_2(s))$, the board chooses cutoff $\hat{s}$ to maximize

$$\int_{\hat{s}}^{1}\left\{\frac{f_1(s)}{f_1(s)+f_0(s)}\left[(1-\lambda)e_1 + \lambda e_2(s)\right](1-w)\right\} d\frac{F_1(s)+F_0(s)}{2} + \frac{F_1(\hat{s})+F_0(\hat{s})}{2}\left[\underline{\pi}(e_1) - k\right], \tag{11}$$

$$\Rightarrow \hat{s}(e_1, e_2(s); w, k) \text{ solves } \frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})}\left[(1-\lambda)e_1 + \lambda e_2(\hat{s})\right](1-w) = \underline{\pi}(e_1) - k. \tag{12}$$

As in the case where $\lambda = 0$, the board chooses a cutoff such that the expected profit created by the marginal incumbent manager is equal to the expected profit under replacement.

Given contract (w, k), the equilibrium cutoff and effort profile are pinned down by equations (8), (9), and (12).[20] Alternatively, we can calculate the corresponding severance pay k and effort decision $(e_1, e_2(s))$ given the wage rate and board's desired equilibrium cutoff as follows:

$$e_1(\hat{s}, w) = \frac{1 - F_1(\hat{s})}{2}(1 - \lambda)w, \tag{13}$$

$$e_2(s, \hat{s}, w) = \frac{f_1(s)}{f_1(s) + f_0(s)}\lambda w \text{ if } s \geq \hat{s}, \tag{14}$$

and

$$k(\hat{s}, w) = \underline{\pi}(e_1(\hat{s}, w)) - \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}\left[(1 - \lambda)e_1(\hat{s}, w) + \lambda e_2(\hat{s}, \hat{s}, w)\right](1 - w). \tag{15}$$

Analysis of the first-stage game

The board maximizes expected profit over $(\hat{s}, w)$, with $e_1(\hat{s}, w)$, $e_2(s, \hat{s}, w)$, and $k(\hat{s}, w)$ as determined in equations (13), (14), and (15), respectively. Substituting equations (13), (14), and (15) into the board's profit function (11) yields expected profit as a function of $(\hat{s}, w)$.

$$\begin{aligned}\pi(\hat{s}, w) :=& w(1 - w) \times \left\{(1 - \lambda)^2\left(\frac{1 - F_1(\hat{s})}{2}\right)^2 + \lambda^2\int_{\hat{s}}^1\left(\frac{f_1(s)}{f_1(s) + f_0(s)}\right)^2 d\frac{F_1(s) + F_0(s)}{2}\right.\\ &\left. + \frac{F_1(\hat{s}) + F_0(\hat{s})}{2} \times \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \times \left[(1 - \lambda)^2\frac{1 - F_1(\hat{s})}{2} + \lambda^2\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}\right]\right\}.\end{aligned} \tag{16}$$

Therefore, the board's problem reduces to maximization of $\pi(\hat{s}, w)$ subject to the limit liability constraint $k(\hat{s}, w) \geq 0$.

5.2 | Equilibrium replacement policy under optimal contract

Next we investigate the equilibrium replacement policy under the optimal contract. For the ease of our exposition, we proceed by ignoring the nonnegativity constraint for severance pay.[21] It is straightforward to verify that $w^* = \frac{1}{2}$ under the optimal contract. Therefore, the expected profit can be rewritten in terms of $\hat{s}$ alone, and can be usefully decomposed into five components as follows:

$$\begin{aligned}\pi\left(\hat{s}, \frac{1}{2}\right) =& \frac{1}{4} \times \left\{(1 - \lambda)^2 \underbrace{\frac{1 - F_1(\hat{s})}{2}}_{\text{incentive effect}} + \lambda^2 \underbrace{\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}}_{\text{learning effect}}\right\}\\ &\times \left\{\underbrace{\left[\frac{1}{2}\left[1 - F_1(\hat{s})\right] + \frac{1}{4}\left[F_1(\hat{s}) + F_0(\hat{s})\right]\right]}_{\text{selection effect}} + \underbrace{\frac{1}{2}\left[F_1(\hat{s}) + F_0(\hat{s})\right]\left(\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} - \frac{1}{2}\right)}_{\text{commitment cost effect}}\right\}\\ &+ \underbrace{\lambda^2\int_{\hat{s}}^1\left[\left(\frac{f_1(s)}{f_1(s) + f_0(s)} - \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}\right) \times \frac{f_1(s)}{f_1(s) + f_0(s)}\right] d\frac{F_1(s) + F_0(s)}{2}}_{\text{sorting effect}}.\end{aligned} \tag{17}$$

As we mentioned, the *learning effect* and the *sorting effect* enter the expression of the board's profit function if the probability of success depends on period-2 effort (i.e., $\lambda > 0$). The next proposition reports results that are parallel to those in Proposition 1.

Proposition 2. *Suppose that* $\{f_1(\cdot;\alpha), f_0(\cdot;\alpha)\}$ *satisfies Assumptions 1–4. Then,*

(i) *anti-entrenchment emerges in the optimal contract if* α *is sufficiently large for all* $\lambda \in (0, 0.2231]$;[22]

(ii) *entrenchment emerges in the optimal contract if* α *is sufficiently small for all* $\lambda \in (0, 1]$.

The intuition for entrenchment to emerge under the optimal contract as $\alpha \to 0$ is as follows. Recall that when the information structure becomes sufficiently uninformative, the estimate of the incumbent manager's ability at *any* signal $s \in (0, 1)$ is very close to the prior. As a result, the learning effect is the same for all signals, and the sorting effect vanishes because receiving a signal above the cutoff signal $\hat{s}$ reveals little additional information about the incumbent's ability. In this case, the two new effects are not decisive in determining the equilibrium cutoff in the optimal contract; and entrenchment emerges as predicted in Proposition 1 as $\alpha \to 0$.

The intuition is less straightforward when the information structure becomes sufficiently informative. In fact, optimizing the learning effect leads to an equilibrium cutoff greater than $\frac{1}{2}$ while optimizing the sorting effect leads to an equilibrium cutoff below $\frac{1}{2}$. To see this, let us first consider a contract that induces the first-best cutoff $\frac{1}{2}$. In equilibrium, the incumbent manager's posterior belief about his ability when he observes the cutoff signal is equal to the prior. An alternative contract that induces an equilibrium cutoff signal slightly above $\frac{1}{2}$ allows the incumbent manager on the margin to be almost sure that he is of high ability and he will double his period-2 effort from $\frac{1}{2}\lambda w$ to λw [see equation (14)]. Therefore, by the learning effect alone, the board has incentive to induce anti-entrenchment. On the other hand, because the manager is very certain about his type for $s \neq \frac{1}{2}$, a signal that is strictly above the first-best cutoff conveys little additional information about his ability. Therefore, although the incumbent manager can condition his period-2 effort on the signal, the corresponding effort level does not vary as much and the sorting effect vanishes by inducing anti-entrenchment. When the period-1 effort is more important than the period-2 effort in determining the outcome, that is, when λ is small (i.e., $\lambda < 0.2231$), both the learning effect and the sorting effect become less important than the other three effects in expression (17) when the board designs the contract. Therefore, anti-entrenchment emerges when the information structure becomes completely informative as in Proposition 1.

5.3 | Numerical illustration

Propositions 1 and 2 give no indication whether entrenchment or anti-entrenchment is optimal for moderate values of α and $\lambda \in [0.2231, 1)$. In particular, the optimal period-2 effort implemented is zero if the incumbent manager is replaced for the case where $\lambda = 0$, which in turn implies the optimality of zero wage to the replacement manager. Therefore, one may conclude that anti-entrenchment relies on the assumption that the period-2 effort is less crucial than the period-1 effort in determining the firm's success (i.e., $\lambda < 0.2231$), or is due to the fact that the optimal period-2 wage to the replacement manager is (almost) zero. In fact, what really allows anti-entrenchment to arise is the board's incentive to reset the wage (not necessarily down to zero); it is also possible for anti-entrenchment to emerge under the optimal contract when the replacement manager's effort and wage are both positive. Next, we parameterize the information structure as described in Example 1 and provide a numerical example to elaborate on this point.

Example 2. Let $\lambda = \frac{1}{2}$, that is, the effort levels exerted in both periods are equally important to the firm's success. Next, we set $\alpha = 1$, implying that $f_1(s) = 2s$ and $f_2(s) = 2(1 - s)$ for all $s \in [0, 1]$. The optimal contract to the incumbent manager is $(w^*, k^*) \simeq (0.5314, 0)$, and the wage to the replacement manager is $w_r^* \simeq 0.3104 > 0$. The resulting equilibrium replacement cutoff signal is $\hat{s}^* \simeq 0.5352$ and thus anti-entrenchment arises in equilibrium.

Regarding the equilibrium variables, it follows from equations (8) and (9) that the incumbent manager's equilibrium period-1 effort is $e_1^* = \frac{1-(\hat{s}^*)^2}{2}(1-\lambda)w \simeq 0.0948$ and the equilibrium period-2 effort is $e_2^*(s) = \lambda w^* s \simeq 0.2657s$ for $s \geq \hat{s}$. Similarly, the replacement manager's equilibrium effort level is $e_2^{r*} = \frac{1}{2}\lambda w_r^* \simeq 0.0776$ from equation (6), which is nonzero and is close to e_1^* when $\lambda = \frac{1}{2}$.

Figure 3 illustrates the equilibrium replacement policy under the optimal contract in the (α, λ) space.[23] The region between the two dotted curves in Figure 3 depicts the combination of (α, λ) for which anti-entrenchment emerges. The region of (α, λ) to the left of the lower dotted curve depicts the combination of (α, λ) for which entrenchment emerges under the optimal contract. Similarly, the region of (α, λ) to the right of the upper dotted curve depicts the combination of (α, λ) for which the equilibrium replacement cutoff coincides with the first-best replacement cutoff ($\hat{s}^* = \frac{1}{2}$).

The first pattern to notice is that fixing λ, anti-entrenchment (respectively, entrenchment) emerges when information structure is sufficiently informative (respectively, uninformative). This confirms the result in Proposition 2. The second pattern to notice

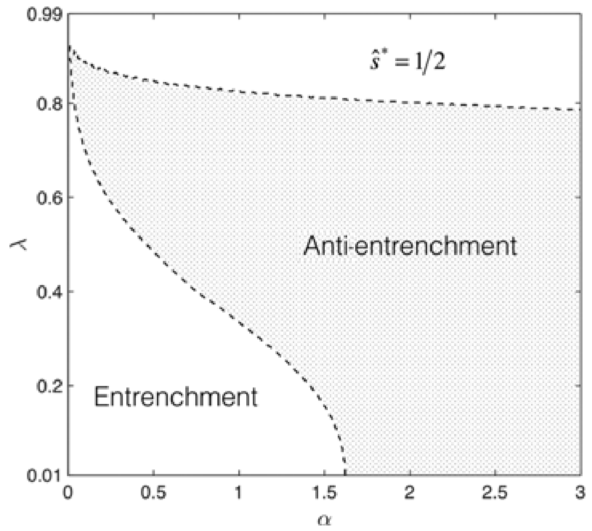


FIGURE 3 Equilibrium replacement policy under optimal contract in the (α, λ) space

is that neither entrenchment nor anti-entrenchment is observed when λ is sufficiently large. The intuition is as follows. Consider the extreme case where $\lambda \to 1$, in which the success probability reduces to $\tilde{\theta}_2 \tilde{e}_2$ and the period-1 effort has almost no impact in determining the outcome. Therefore, the board faces no trade-off between increasing period-1 effort and optimizing ability selection, and will accordingly design a contract to induce the equilibrium signal cutoff that is equal to $\frac{1}{2}$.

6 | DISCUSSIONS AND EXTENSIONS

6.1 | The role of key modeling assumptions

Thus far, we have assumed that: (i) severance pay can be used in the design of the optimal contract; (ii) severance pay is constant with respect to outcome, that is, the board cannot provide performance-based severance pay; (iii) the signal is noncontractible. In this section, we briefly discuss the role of each assumption in shaping the equilibrium cutoff under the optimal contract.

The role of severance pay

Suppose that severance pay is ruled out in the design of the optimal contract, or equivalently, that the board is constrained to use $k = 0$. Then we can show that anti-entrenchment always arises in equilibrium. To see this, first notice that by the standard argument as in the previous pages, it can be shown that $w^* = \frac{1}{2}$ in the optimal contract. Second, fixing $e_1 > 0$, the board's indifference condition can be easily obtained by letting $k = 0$ in equation (12):

$$\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \left[(1-\lambda)e_1 + \lambda e_2(\hat{s})\right](1 - w^*) = \underline{\pi}(e_1).$$

The left-hand side of the above equality is the board's expected profit if the incumbent is retained; and the right-hand side is the board's expected profit from replacement. Recall from the analysis in Section 5.1 that it is optimal for the board to offer $w_r(e_1) = \max\{\frac{1}{2} - \frac{1-\lambda}{\lambda^2} e_1, 0\}$ to the replacement manager. In other words, if the board receives a signal $s = \frac{1}{2}$ and the incumbent manager exerts period-1 effort $e_1 > 0$, the board would then like to lower the wage from $w^* = \frac{1}{2}$ to $w_r(e_1) < \frac{1}{2}$ to further increase its profit. However, it is not allowed to reset the period-1 wage unless it fires the incumbent manager! This gives the board incentive to replace the incumbent manager even if the signal reveals that ability of the incumbent is equal to that of a replacement (i.e., $s = \frac{1}{2}$). More formally, fixing $e_1 > 0$, retaining the incumbent manager after observing signal $s = \frac{1}{2}$ generates expected profit

$$\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \left[(1-\lambda)e_1 + \lambda e_2(\hat{s})\right](1 - w^*) = \frac{1}{4}\left[(1-\lambda)e_1 + \frac{1}{4}\lambda^2\right]$$

while it follows from (7) that replacing the manager yields expected profit

$$\underline{\pi}(e_1) = \begin{cases} \frac{1}{4}\left(\frac{1}{2}\lambda + \frac{1-\lambda}{\lambda}e_1\right)^2 & \text{for } e_1 \leq \frac{1}{2}\frac{\lambda^2}{1-\lambda}, \\ \frac{1}{2}(1-\lambda)e_1 & \text{for } e_1 > \frac{1}{2}\frac{\lambda^2}{1-\lambda}. \end{cases} \tag{18}$$

It is straightforward to verify that $\frac{1}{4}[(1-\lambda)e_1 + \frac{1}{4}\lambda^2] < \underline{\pi}(e_1)$ for all $e_1 > 0$, that is, anti-entrenchment always emerges in equilibrium if severance pay cannot be utilized.

The fact that anti-entrenchment can emerge in equilibrium stems from the observation that the board has incentive to modify the wage after the incumbent manager exerts effort in the first period. Together with the result in Proposition 2, it follows instantly that when the board's information structure is not informative, it can benefit by lowering the equilibrium standard, which severance allows.

Outcome-dependent severance pay

Note that severance pay is assumed constant with respect to outcome in the model, that is, the board cannot provide performance-based severance pay. In practice, a standard severance contract consists of a one-time cash component and an equity element, such as vesting in stock and options.[24] Next we show that the conflict between the ability selection problem and the period-1 moral hazard problem can be alleviated if the board can condition the severance pay on the outcome.

A contract to the incumbent manager is now in the form of a triple (w_1, w_2, k). The variable w_1 is the wage rate when the incumbent manager stays as in the baseline model. The tuple (w_2, k) constitutes a severance package, where w_2 is the payment to the incumbent manager if he is forced out and $y = 1$. Again, the variable k is the constant severance pay as in the baseline model.

Proposition 3. *Suppose that the signal is noncontractible and the board can provide performance-based severance pay. Then* $k^* = 0$ *in the optimal contract for all* $\lambda \in [0, 1)$. *Moreover, if* $\lambda = 0$, *then* $w_1^* = w_2^*$ *and* $\hat{s}^* = \frac{1}{2}$.

Proposition 3 states that the lump-sum severance pay is always zero in the optimal contract. This result is intuitive. Constant severance pay is less effective to the board than performance-based severance pay when the incumbent manager is no longer in office because a lump-sum payment rewards failure. In other words, the agency problem is more severe for firms that use severance contract containing a cash component. Therefore, only performance-based severance pay is employed in the optimal contract.

In the extreme case where $\lambda = 0$, the board has no incentive to deviate from the first-best cutoff. Due to the manager's risk neutrality, the effort choice of the incumbent manager is determined only by the expected wage. For a given effort level e_1 that the board would like to induce, the expected wage, which is also the total cost of hiring the incumbent manager, is fixed. Because e_1 is fixed, it remains only to maximize the expected ability of the manager who stays in office in period 2. Hence, the replacement cutoff stays at the first best in the optimal contract.[25]

Contractibility of signal

If the board's signal is contractible, a contract to the incumbent manager is fully characterized by $\{w(s), r(s), k(s)\}$ for each possible signal $s \in [0, 1]$, where $\{w(s), k(s)\}$ is the promised wage and severance pay, and $r(s) \in [0, 1]$ specifies the probability that the incumbent manager will be retained at signal s. In particular, $r(s) = 1$ indicates that the incumbent manager is retained whereas $r(s) = 0$ indicates that the incumbent is fired with certainty.[26] The contract with the replacement manager is again denoted by w_r.

Proposition 4. *Suppose that the signal is contractible and severance pay is constant with respect to outcome. Then* $k^*(s) = 0$ *in the optimal contract for all* $\lambda \in [0, 1)$. *Moreover,* $r^*(s) = 1$ *for* $s \in [\frac{1}{2}, 1]$ *and* $r^*(s) = 0$ *for* $s \in [0, \frac{1}{2})$.

To explain the intuition most cleanly, let us consider the case where $\lambda = 0$. Then the probability of success degenerates to $\tilde{\theta}_2 e_1$. First, note that allowing the board to contract on signals gives the board commitment power on its retention decision at no cost. Because severance pay is a costly commitment device, it is no longer used in the optimal contract. Second, note that the board can design a contract to induce any effort level without deviating from the socially optimal replacement cutoff. Because the incumbent manager is risk-neutral and only cares about the expected wage, the board can motivate the incumbent manager by increasing the expected wage payment when $y = 1$, which is determined by both the wage function $w(s)$ and the replacement policy $r(s)$. This in turn implies that fixing effort e_1 and a replacement policy $r(s)$, the board can adjust the wage function $w(s)$ to induce e_1 without changing $r(s)$. In other words, the board can optimize effort and ability selection separately if the signal is

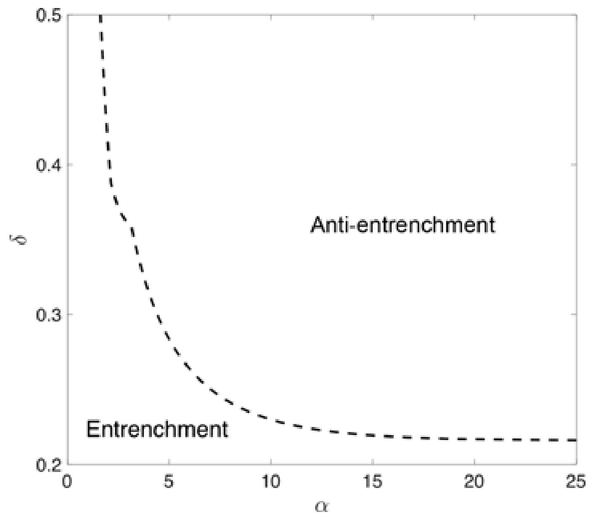


FIGURE 4 Equilibrium replacement policy under optimal contract in the (α, δ) space

contractible. Therefore, the board has no incentive to distort ability selection and the replacement cutoff in the optimal contract is always $\frac{1}{2}$.

6.2 | Extensions

In this section, we show that the main results derived in Propositions 1 and 2 are robust to several different specifications. All the results in this section are described only informally in the text; the formal models and analyses are in Online Appendix B.

Variance of incumbent manager's ability

We first generalize the success function and investigate how the volatility of the incumbent manager's ability affects our main results on entrenchment and anti-entrenchment. For simplicity, we assume that $\lambda = 0$ and thus the probability of success in expression (1) degenerates to $\tilde{\theta}_2 e_1$.[27] To investigate the impact of ability volatility, we assume that the ability space is $\theta \in \{\theta_L, \theta_H\} \equiv \{\frac{1}{2} - \delta, \frac{1}{2} + \delta\}$, where $\delta \in (0, \frac{1}{2}]$. Clearly, the parameter δ measures the *ex ante* variance of the manager's ability, and the baseline model in Section 4 corresponds to $\delta = \frac{1}{2}$. When $\delta < \frac{1}{2}$, we have that $\theta_L > 0$. Fixing $(w, \hat{s})$, the incumbent manager's period-1 effort is

$$e_1 = \frac{1}{2}\left[\left(\frac{1}{2} + \delta\right)\left[1 - F_1(\hat{s})\right] + \left(\frac{1}{2} - \delta\right)\left[1 - F_0(\hat{s})\right]\right] w, \tag{19}$$

from which we can see that the incumbent manager, when choosing his period-1 effort level, takes into consideration that his effort would increase the probability of success even if he is of low ability.[28]

Intuitively, as δ increases, selecting a high-ability manager becomes more important to the board relative to inducing more period-1 effort. As a result, anti-entrenchment is more likely to emerge in equilibrium under the optimal contract for large δ. In Online Appendix B, we show that anti-entrenchment arises as $\alpha \to \infty$ if and only if $\delta > \frac{\sqrt{2}-1}{2} \approx 0.2071$.

Figure 4 depicts the equilibrium replacement policy under the optimal contract in the (α, δ) space using the information structure described in Example 1. The contour plot is shown for $(\alpha, \delta) \in [0, 25] \times [0.2, 0.5]$. The region of (α, δ) below (respectively, above) the dotted curve depicts the combination of (α, δ) for which entrenchment (respectively, anti-entrenchment) emerges under the optimal contract. The downward slope of the contour plot confirms the intuition obtained at the limiting distribution: anti-entrenchment is more likely to emerge for large α and large δ.

Signal of outcome instead of ability

In the baseline model, it is assumed that the board observes a signal of the incumbent manager's ability rather than the outcome under the incumbent's management. Because the signal is not related to the incumbent manager's effort, the incumbent cannot increase his retention probability by exerting more effort. When the board receives a signal related to effort or outcome, the incumbent manager is able to increase his retention probability by exerting more effort. Unlike the baseline model, severance pay now becomes a double-edged sword. On the one hand, it is a costly commitment device that provides job security (as in the baseline model), and hence can motivate more period-1 effort. On the other hand, it increases the value of replacement to the incumbent manager and may give the incumbent incentive to reduce period-1 effort so as to increase the turnover rate.[29] Again, it can be shown that the main results derived in Proposition 1 are robust. The intuition is as follows.

Let us consider the case where the information structure is sufficiently uninformative. Expecting that the board has noisy monitoring technology, the incumbent manager has little incentive to manipulate the realization of the signal. In other words, the aforementioned negative effect of severance pay on the period-1 effort is negligible and the model degenerates to the baseline at the limit. Therefore, entrenchment is expected to emerge under the optimal contract.

When the information structure is sufficiently informative, the negative effect of the severance pay on the period-1 effort is strong. Under this scenario, the board can simply avoid the disadvantage of k by offering low severance pay. Interestingly, this does not contradict the possibility of obtaining a net commitment gain. In fact, we can construct a contract with a high wage and zero severance pay that yields anti-entrenchment and dominates all feasible contracts that induce entrenchment in terms of board's profit. Thus, anti-entrenchment emerges in the optimal contract.

Signal is private information to the board

When the incumbent manager cannot observe the signal, he cannot condition his period-2 effort on it. Therefore, the sorting effect is absent. However, the learning effect still exists but with a different expression. Specifically, fixing an equilibrium cutoff $\hat{s}$, the incumbent manager updates the belief about his ability to

$$\frac{1 - F_1(\hat{s})}{\left[1 - F_1(\hat{s})\right] + \left[1 - F_0(\hat{s})\right]}.$$

It is straightforward to verify that the above expression is greater than $\frac{1}{2}$. Intuitively, the incumbent manager learns from his retention that his ability is above average. Because ability and effort are complements, a higher estimate of ability implies a higher marginal return on effort.[30] Therefore, the incumbent exerts more effort in period 2 than does the potential replacement manager with the same wage. In Online Appendix B.3, we show that the result derived in Proposition 2 is robust: if $\lambda < \sqrt{2} - 1$, entrenchment emerges as $\alpha \to 0$ and anti-entrenchment emerges in equilibrium under optimal contract as $\alpha \to \infty$.

7 | CONCLUSION

This paper explores how the problem of motivating the incumbent manager to exert effort and keeping the flexibility to choose a high-ability manager interacts with the equilibrium replacement policy. We focus on the situation where the board observes a noncontractible signal after the incumbent manager exerts effort and solve for the optimal contract. We show that the information technology that the board uses to assess the incumbent manager's ability is an important determinant of the optimal contract and of managerial turnover. Unlike the existing literature on managerial turnover, which aims to rationalize entrenchment, we show that anti-entrenchment can also be optimal in equilibrium for shareholders in some situations. This result is robust to allowing the signal to be the board's private information and to the possibility that the board observes a signal of the outcome under the incumbent's management rather than a signal of his ability. The model highlights the board's monitoring technology as an important determinant of managerial turnover.

There are several interesting questions that can be pursued using the stylized model introduced in this paper. For future research, it would be interesting to endogenize the informativeness of the board's monitoring technology. In practice, informativeness is often the choice of the board. Some boards actively monitor their CEOs while some tend to be passive monitors. Endogenizing the board's monitoring technology could help us better understand the differences of monitoring intensity that occur across industries.

Another intriguing research avenue would incorporate voluntary departure into the model. In this paper, we assume that the board and the incumbent manager learn the manager's ability symmetrically. In practice, the incumbent manager may privately learn his ability (or the firm's future performance under his management) in the interim stage of a contractual term (e.g., Inderst & Mueller, 2010). In such a scenario, severance pay can be utilized in the optimal contract to provide incentive for the incumbent manager to step down voluntarily whenever necessary. We find the idea of developing a model that includes private information for the manager and allows for both forced and voluntary departure very interesting. We leave the exploration of this possibility for future research.

ENDNOTES

[1] Although evidence shows forced CEO turnover is increasing over time and indicates boards are using more aggressive replacement policies, it is widely believed that CEOs are rarely fired and thus are entrenched. For instance, Kaplan and Minton (2012) find that board-driven turnover increased steadily from 10.93% (1992–1999) to 12.47% (2000–2007) using data from publicly traded Fortune 500 companies.

[2] See Laux (2014) for a comprehensive survey of the theoretical models on this topic.

[3] In order to focus on the period-1 incentive, Almazan and Suarez (2003) assume away the period-2 moral hazard problem. Therefore, there is no need for the board to offer a contract to the replacement manager. This corresponds to our analysis in Section 4.

[4] Interestingly, Inderst and Klein (2007) show that both underinvestment and overinvestment can arise if there exists competition between division managers and only one investment can be undertaken.

[5] We assume $\theta_L = 0$ for the sake of simplicity. In "Variance of incumbent manager's ability" in Section 6.2, we allow θ_L to be nonzero and again show that the main results remain unchanged.

[6] This assumption is necessary because it excludes the possibility that the board sells the whole firm to the manager in order to provide the greatest possible incentive in the optimal contract.

[7] In general, the output function can also depend on the incumbent manager's ability θ_1. Because θ_1 is predetermined when the board designs the contract, all the results naturally hold without adding much economic insight and we drop θ_1 in output function (1) for simplicity.

[8] Efforts are assumed to be substitutes for simplicity (e.g., Lizzeri, Meyer & Persico, 2002). Once complementarity of effort is introduced, period-2 effort will depend on period-1 effort and the model becomes less tractable. On the one hand, the replacement manager needs to form an expectation about e_1 and chooses e_{2r} according to this expectation. On the other hand, the analysis of the incumbent manager's effort choices is complicated by his double deviations, that is, deviations in which the incumbent manager switches to a different effort level in period 1 and reoptimizes his subsequent period-2 effort for the new period-1 effort level.

[9] We assume that severance payment is made only after forced departure. In other words, if the manager quits voluntarily, he does not receive any severance payment. We make this assumption because (i) we focus on the *ex ante* severance agreements and recent evidence suggests that the majority of departing CEOs get paid according to the amount specified in the severance contract (Gillan & Nguyen, 2016; Goldman & Huang, 2015; Yermack, 2006); and (ii) according to Yermack (2006), "A CEO who retires voluntarily in the middle of a contract generally is not entitled to severance unless the board awards it discretionarily or the CEO negotiates to retain his status as an employee."

[10] Both completely informative and completely uninformative information structures are defined using pointwise convergence.

[11] If the incumbent is of low ability, the probability of success is always 0 independent of the period-1 effort e_1.

[12] Here, we highlight the fact that the incumbent's effort choice depends on the contract (w, k) and the board's replacement policy $\hat{s}$.

[13] Again, here we highlight the fact that the board's best response depends on the contract (w, k) as well as the period-1 effort e_1.

[14] The optimal wage to the replacement manager is zero because his effort has no impact on the ultimate outcome when $\lambda = 0$. It is useful to point out that this is not crucial for the anti-entrenchment result. See Section 5.3 for more discussion of zero payment to the replacement manager.

[15] We thank one referee for pointing out this intuition.

[16] To see this more clearly, note that fixing a replacement policy $\hat{s}$: (i) with probability $\frac{1}{2}[1 - F_1(\hat{s})]$, the incumbent manager is of high ability and is retained; (ii) with probability $\frac{1}{2}[1 - F_0(\hat{s})]$, the incumbent manager is of low ability and is retained; (iii) with probability $\frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]$, the incumbent manager is replaced by a manager with expected ability $\mathbb{E}(\theta_r) = \frac{1}{2}$. Therefore, the expected ability of the manager in period 2 is: $\frac{1}{2}[1 - F_1(\hat{s})] \cdot 1 + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})] \cdot 0 + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})] \cdot \frac{1}{2} = \frac{1}{2}[1 - F_1(\hat{s})] + \frac{1}{4}[F_1(\hat{s}) + F_0(\hat{s})]$.

[17] Here we highlight the fact that the equilibrium cutoff under the optimal contract depends on the informativeness of the board's information structure α in our notation.

[18] Recently, the concept of ρ-concavity has been applied to the study of different topics in economics. See Mares and Swinkels (2014) on auction theory; and Anderson and Renault (2003), Weyl and Fabinger (2013) on industry organization. To the best of our knowledge, this is the first paper to use ρ-concavity to define information order.

[19] It is useful to point out that the incumbent cannot adjust his period-2 effort according to the signal, and the sorting effect is absent if the signal is the board's private information. We will discuss this situation in Section 6.2.

[20] It is assumed that the equilibrium most favorable to the board is selected when multiple equilibria exist.

[21] The limited liability constraint for severance pay is rigorously considered in the proof of Proposition 2.

[22] The threshold of 0.2231 is obtained in the following two steps. First, for all $\hat{s} \in [0, \frac{1}{2}]$, we show in the Appendix that the board's expected profit can be bounded above by $\frac{3}{32}[(1-\lambda)^2 + \lambda^2] + \frac{1}{4}\lambda^2 \int_0^1 [f_1(s;\alpha)]^2 ds$, where the second term approaches $\frac{1}{2}\lambda^2$ as $\alpha \to \infty$. Second, we construct a tuple $(\hat{s}, w) = (\frac{1}{2} + \kappa, \frac{(1-\hat{s})+2\psi}{2(1-\hat{s})+2\psi})$, where $\psi \equiv (\frac{\lambda}{1-\lambda})^2$ and $\kappa > 0$, and show that there exists a sufficiently small $\kappa > 0$ such that the constructed tuple satisfies the limited liability constraint $k(\hat{s}, w) \geq 0$ and generates more profit than $\frac{3}{32}[(1-\lambda)^2 + \lambda^2] + \frac{1}{2}\lambda^2$ as α becomes sufficiently large if $\psi < \frac{\sqrt{233}-9}{76}$, or equivalently, $\lambda < \sqrt{\frac{\sqrt{233}-9}{76}}/[1 + \sqrt{\frac{\sqrt{233}-9}{76}}] \approx 0.2231$.

[23] The contour plots are shown for $(\alpha, \lambda) \in [0, 3] \times [0.01, 0.99]$.

[24] According to Huang (2011), about 40% of the severance contracts issued by S&P 500 firms from 1993 through 2007 contain only a cash component while 60% of them also include an equity component.

[25] It is useful to point out that a severance contract that contains only an equity element is rarely observed in practice. A severance contract with a one-time cash component can be rationalized by the assumption that the manager is more risk-averse than the board. Under this scenario, the optimal contract may involve some degree of lump-sum payment in response to risk sharing.

[26] Due to the board's risk neutrality, randomization is not optimal except in the case where the board is indifferent between retaining and firing the incumbent manager. In such a case, we assume that the incumbent manager is retained with probability 1.

[27] In Online Appendix B, we further generalize the firm's success function to $\tilde{\theta}_2 e_1^{1+\tau}$, where $\tau \in (-1, 1)$ measures the relative importance of effort compared to ability selection. Again, we show that the main results on entrenchment and anti-entrenchment are robust.

[28] Equation (19) degenerates to equation (2) at $\delta = \frac{1}{2}$.

[29] Note that in this extension the equilibrium cutoff influences agent's period-1 effort choice as in the baseline model in Section 4. In addition, the conjectured period-1 effort will also influence the board's equilibrium decision on the replacement cutoff. This economic insight is in some sense similar to that in Taylor and Yildirim (2011). They study the benefits and costs of different review policies and identify conditions under which the evaluator commits not to utilize the agent's information and chooses blind review as the optimal policy. The main issue in their model is that the evaluator's prior belief about the quality of the project is endogenous, and this belief is determined by the agent's effort. Therefore, the standard influences the effort and the conjectured effort in turn influences the standard.

[30] It is useful to point out that the incumbent is easier to motivate than the replacement as in Casamatta and Guembel (2010) but for a different reason. Specifically, the incumbent manager is easier to motivate in Casamatta and Guembel (2010) due to his reputational concern, whereas in our model the ease of his motivation is due to his awareness of his ability and the complementarity between ability and effort.

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APPENDIX: PROOFS OF THE PROPOSITIONS

Proof of Lemma 1.

Proof. First, it is straightforward to verify that $e_1^{\mathcal{FB}} = \frac{1}{2}(1-\lambda)[1+\frac{1}{2}[F_0(\hat{s}^{\mathcal{FB}}) - F_1(\hat{s}^{\mathcal{FB}})]]$ and $e_{2r}^{\mathcal{FB}} = \frac{1}{2}\lambda$. Moreover, fixing $\hat{s} = \hat{s}^{\mathcal{FB}}$, pointwise optimization for each possible signal implies instantly that for $s \geq \hat{s}$, the board enforces effort $e_2^{\mathcal{FB}}(s) = \frac{f_1(s)}{f_1(s)+f_0(s)}\lambda$.

Second, the first-order condition with respect to $\hat{s}$ yields

$$\frac{f_1(\hat{s}^{\mathcal{FB}})}{f_1(\hat{s}^{\mathcal{FB}}) + f_0(\hat{s}^{\mathcal{FB}})}\left[(1-\lambda)e_1^{\mathcal{FB}} + \lambda e_2^{\mathcal{FB}}(\hat{s}^{\mathcal{FB}})\right] - \frac{1}{2}\left[e_2^{\mathcal{FB}}(\hat{s}^{\mathcal{FB}})\right]^2 = \frac{(1-\lambda)e_1^{\mathcal{FB}} + \lambda e_{2r}^{\mathcal{FB}}}{2} - \frac{1}{2}\left[e_{2r}^{\mathcal{FB}}\right]^2.$$

Carrying out the algebra, the above condition can be further simplified as

$$\frac{f_1(\hat{s}^{\mathcal{FB}})}{f_1(\hat{s}^{\mathcal{FB}}) + f_0(\hat{s}^{\mathcal{FB}})} = \frac{1}{2}.$$

The above equality, together with Assumptions 1 and 3, implies that $\hat{s}^{\mathcal{FB}} = \frac{1}{2}$. This completes the proof. □

Proof of Proposition 1.

Proof. It is without loss of generality to normalize the information structure such that $s = \frac{1}{2}F_1(s) + \frac{1}{2}F_0(s)$ for $s \in [0,1]$, which in turn implies that $f_1(s) + f_0(s) = 2$ for $s \in [0,1]$. Moreover, Assumption 1 implies that $f_1(s)$ is strictly increasing in s; Assumption 2 implies that $f_1(0) = 0$ and $f_1(1) = 2$; and Assumption 3 implies that $f_1(\frac{1}{2}) = 1$ and $F_1(s) + F_0(1-s) = 1$ for all $s \in [0,1]$.

The proof proceeds in two steps. First, we prove uniform convergence of $F_1(\cdot)$ as $\alpha \to \infty$ and $\alpha \to \infty$, and derive the corresponding limiting distributions in Lemmas A1 and A2, respectively. Second, based on the established properties of $F_1(\cdot)$, we prove the anti-entrenchment (respectively, entrenchment) result by constructing a contract that induces an equilibrium cutoff greater (respectively, less) than $\frac{1}{2}$ and generates a higher profit than does any contract that induces entrenchment (respectively, anti-entrenchment) as α becomes sufficiently large (respectively, small). □

Lemma A1 (Uniform Convergence of $F_1(\cdot)$ as $\alpha \to \infty$). *For any given $\epsilon > 0$, there exists a threshold N_a such that for $\alpha > N_a$, $F_1(s;\alpha) < \epsilon$ for all $s \in [0, \frac{1}{2}]$ and $F_1(s;\alpha) < (2s-1)+\epsilon$ for all $s \in [\frac{1}{2}, 1]$.*

Proof. It follows from the definition of the completely informative information structure that, given $\frac{1}{2}\epsilon$, there exists a threshold N_a such that

$$f_1\left(\frac{1}{2}(1-\epsilon);\alpha\right) < \frac{1}{2}\epsilon, \text{ for } \alpha > N_a. \tag{A.1}$$

Therefore, we have that

$$\begin{aligned} F_1\left(\frac{1}{2};\alpha\right) = \int_0^{\frac{1}{2}} f_1(t;\alpha)dt &= \int_0^{\frac{1}{2}(1-\epsilon)} f_1(t;\alpha)dt + \int_{\frac{1}{2}(1-\epsilon)}^{\frac{1}{2}} f_1(t;\alpha)dt \\ &\leq \frac{1}{2}(1-\epsilon)\frac{1}{2}\epsilon + \left[\frac{1}{2} - \frac{1}{2}(1-\epsilon)\right] < \epsilon, \end{aligned}$$

where the first inequality follows from (A.1) and $f_1(s) \leq 1$ for $s \in [0, \frac{1}{2}]$. It follows immediately from the above inequality that

$$F_1(s;\alpha) \leq F_1\left(\frac{1}{2};\alpha\right) < \epsilon, \text{ for all } s \in \left[0, \frac{1}{2}\right].$$

Similarly, for all $s \in [\frac{1}{2}, 1]$, we have that

$$F_1(s;\alpha) = 2s - F_0(s;\alpha) = (2s-1) + F_1(1-s;\alpha) < (2s-1) + \epsilon, \text{ for } \alpha > N_a,$$

where the first equality follows from the normalization that $\frac{1}{2}[F_1(s;\alpha) + F_0(s;\alpha)] = s$ for all $s \in [0, 1]$ and the second equality follows from Assumption 3. This completes the proof. □

Lemma A2 (Uniform Convergence of $F_1(\cdot)$ as $\alpha \to 0$). *For any given $\epsilon > 0$, there exists a threshold N_e such that for $\alpha < N_e$, $F_1(s;\alpha) > s - \epsilon$ for all $s \in [0, 1]$.*

Proof. It follows directly from the definition of the completely uninformative information structure that fixing $\epsilon > 0$, there exists a threshold N_e such that

$$f_1\left(\frac{1}{2}\epsilon;\alpha\right) > 1 - \epsilon, \text{ for } \alpha < N_e. \tag{A.2}$$

Thus, we have that

$$\begin{aligned} s - F_1(s;\alpha) = \int_0^s \left[1 - f_1(t;\alpha)\right] dt &\leq \int_0^{\frac{1}{2}} \left[1 - f_1(t;\alpha)\right] dt \\ &= \int_0^{\frac{1}{2}\epsilon} \left[1 - f_1(t;\alpha)\right] dt + \int_{\frac{1}{2}\epsilon}^{\frac{1}{2}} \left[1 - f_1(t;\alpha)\right] dt \\ &\leq \frac{1}{2}\epsilon + \epsilon\left(\frac{1}{2} - \frac{1}{2}\epsilon\right) < \epsilon, \text{ for all } s \in \left[0, \frac{1}{2}\right], \end{aligned} \tag{A.3}$$

where the first inequality follows from $f_1(t;\alpha) \leq f_1(\frac{1}{2};\alpha) = 1$ for all $t \in [0, \frac{1}{2}]$, and the second inequality follows from (A.2) and the fact that $1 - f_1(t;\alpha) \leq 1$. For all $s \in [\frac{1}{2}, 1]$, we have that

$$s - F_1(s;\alpha) = s - \left[2s - F_0(s;\alpha)\right] = (1-s) - F_1(1-s;\alpha) < \epsilon,$$

where the second equality follows from $F_1(1-s;\alpha) + F_0(s;\alpha) = 1$ and the inequality follows from (A.3). This completes the proof. □

Now we can prove Proposition 1. Recall that the board's expected profit is

$$\pi(\hat{s};\alpha) = \frac{1}{8}\left[1 - F_1(\hat{s};\alpha)\right]\left\{\frac{1}{2}\left[1 - F_1(\hat{s};\alpha)\right] + \frac{1}{2}\left[F_1(\hat{s};\alpha) + F_0(\hat{s};\alpha)\right]\frac{f_1(\hat{s};\alpha)}{f_1(\hat{s};\alpha) + f_0(\hat{s};\alpha)}\right\}.$$

Together with the fact that $f_0(s;\alpha) = f_1(1-s;\alpha)$, $f_1(s;\alpha) + f_0(s;\alpha) = 2$, and $F_0(s;\alpha) = 1 - F_1(1-s;\alpha)$ for all $s \in [0, 1]$, the expected profit function can be simplified as follows:

$$\pi(\hat{s};\alpha) = \frac{1}{16}\left[1 - F_1(\hat{s};\alpha)\right]\left\{\left[1 - F_1(\hat{s};\alpha)\right] + \hat{s}f_1(\hat{s};\alpha)\right\}.$$

Anti-Entrenchment Note that $\pi(\hat{s};\alpha)$ can be bounded above by

$$\pi(\hat{s};\alpha) < \frac{1}{16}(1+\hat{s}) < \frac{3}{32} \text{ for all } \hat{s} \in \left[0, \frac{1}{2}\right].$$

The first inequality follows from the fact that $1 - F_1(\hat{s};\alpha) < 1$ and $f_1(\hat{s};\alpha) < f_1(\frac{1}{2};\alpha) \equiv 1$ for all $\hat{s} \in [0, \frac{1}{2}]$, and the second inequality follows from the postulated $\hat{s} \in [0, \frac{1}{2}]$.

To complete the proof for the case of anti-entrenchment, it suffices to construct a cutoff strictly above $\frac{1}{2}$ that generates a profit more than $\frac{3}{32}$. It follows from Lemma A1 that fixing $\epsilon > 0$, there exists a threshold N_a such that for $\alpha > N_a$, $1 - F_1(\hat{s};\alpha) >$

$2 - 2\hat{s} - \epsilon$ for all $\hat{s} \in (\frac{1}{2}, 1)$. Moreover, given $\hat{s} \in (\frac{1}{2}, 1)$ and $\epsilon' := \frac{\epsilon}{1+\epsilon} > 0$, Assumption 4 implies that there exists a threshold N_a' such that $\frac{1}{2} f_1(\hat{s}) = \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} > 1 - \epsilon'$ for $\alpha > N_a'$. Let $\overline{\alpha} := \max\{N_a, N_a'\}$. It follows immediately that

$$\pi(\hat{s}; \alpha) > \frac{1}{16}(2 - 2\hat{s} - \epsilon)\left[(2 - 2\hat{s} - \epsilon) + 2\hat{s}(1 - \epsilon')\right], \text{ for } \alpha > \overline{\alpha}.$$

Therefore, the expected profit under cutoff $\frac{1}{2}(1 + \epsilon)$ can be bounded below by

$$\pi\left(\frac{1}{2}(1 + \epsilon); \alpha\right) > \frac{1}{16}(1 - 2\epsilon)\left[(1 - 2\epsilon) + (1 + \epsilon)(1 - \epsilon')\right] = \frac{1}{8}(1 - 2\epsilon)(1 - \epsilon),$$

and it remains to find ϵ such that

$$\frac{1}{8}(1 - 2\epsilon)(1 - \epsilon) \geq \frac{3}{32}.$$

It can be verified that the above inequality holds if $\epsilon \leq \frac{3-\sqrt{7}}{4}$. This completes the proof of anti-entrenchment.

Entrenchment It suffices to show that there exists a threshold $\underline{\alpha}$ such that for $\alpha < \underline{\alpha}$, $\pi(\hat{s}; \alpha) < \pi(0; \alpha) \equiv \frac{1}{16}$ for all $\hat{s} \in [\frac{1}{2}, 1]$. Note that fixing any $\Delta \in (0, \frac{1}{2})$, we have that

$$1 - F_1(1 - \Delta; \alpha) = F_0(\Delta; \alpha) = \int_0^{\Delta} f_0(t)dt < 2\Delta,$$

which in turn implies that

$$\begin{aligned}
\pi(\hat{s}; \alpha) &= \frac{1}{16}\left[1 - F_1(\hat{s}; \alpha)\right]\left\{\left[1 - F_1(\hat{s}; \alpha)\right] + \hat{s} f_1(\hat{s}; \alpha)\right\} \\
&\leq \frac{1}{8}(1 - \hat{s})\left[2(1 - \hat{s}) + 2\right] \\
&\leq \frac{1}{4}\Delta(\Delta + 1), \text{ for all } \hat{s} \in [1 - \Delta, 1].
\end{aligned}$$

Let $\Delta := \frac{\sqrt{2}-1}{2}$. The right-hand side of the above inequality is equal to $\frac{1}{16}$. This in turn implies that $\hat{s} \in [1 - \Delta, 1]$ cannot be the equilibrium replacement policy under optimal contract, and hence it remains to show that there exists a threshold $\underline{\alpha}$ such that for $\alpha < \underline{\alpha}$, $\pi(\hat{s}; \alpha) < \pi(0; \alpha) \equiv \frac{1}{16}$ for all $\hat{s} \in [\frac{1}{2}, 1 - \Delta]$. Note that from the definition of the completely uninformative information structure, fixing any $\epsilon' > 0$, there exists a threshold N_e' such that

$$\frac{1}{2} f_1(\hat{s}) \equiv \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} < \frac{1}{2} + \epsilon', \text{ for all } \hat{s} \in \left[\frac{1}{2}, 1 - \Delta\right] \text{ and } \alpha < N_e'. \tag{A.4}$$

Moreover, it follows from Lemma A2 that, fixing $\epsilon > 0$, there exists a threshold N_e such that

$$F_1(\hat{s}; \alpha) > \hat{s} - \epsilon, \text{ for all } \hat{s} \in [0, 1] \text{ and } \alpha < N_e. \tag{A.5}$$

Let $\epsilon' = \epsilon = \frac{\sqrt{3}}{3} - \frac{1}{2}$ and $\underline{\alpha} = \min\{N_e, N_e'\}$. Then the expected profit for $\alpha < \underline{\alpha}$ can be bounded above by

$$\begin{aligned}
\pi(\hat{s}) &\equiv \frac{1}{16}\left[1 - F_1(\hat{s}; \alpha)\right]\left\{\left[1 - F_1(\hat{s}; \alpha)\right] + \hat{s} f_1(\hat{s}; \alpha)\right\} < \frac{1}{16}(1 - \hat{s} + \epsilon)\left[(1 - \hat{s} + \epsilon) + (1 + 2\epsilon')\right] \\
&= \frac{1}{16}(1 - \hat{s} + \epsilon)(2 - \hat{s} + 3\epsilon) \\
&\leq \frac{3}{16}\left(\frac{1}{2} + \epsilon\right)^2 = \frac{1}{16}, \text{ for all } \hat{s} \in \left[\frac{1}{2}, 1 - \Delta\right],
\end{aligned}$$

where the first inequality follows from (A.4), (A.5), and $\hat{s} \leq 1$; the second inequality follows from $\hat{s} \geq \frac{1}{2}$; and the last equality follows from the postulated $\epsilon = \frac{\sqrt{3}}{3} - \frac{1}{2}$. This completes the proof.

Proof of Proposition 2.

Proof. Define $\hat{S}(\alpha)$ as the set of the cutoff signals that can be induced in equilibrium at wage $w = \frac{1}{2}$ without violating the nonnegativity constraint for k, that is,

$$\hat{S}(\alpha) := \left\{ \hat{s} \,\middle|\, k\left(\hat{s}, \frac{1}{2}; \alpha\right) \geq 0 \,\&\, \hat{s} \in [0,1] \right\}.$$

If $\hat{s} \in \hat{S}(\alpha)$, the board's expected profit can be written as

$$\bar{\pi}(\hat{s}; \alpha) := \pi\left(\hat{s}, \frac{1}{2}; \alpha\right) \equiv \frac{1}{16}\left[(1-\lambda)^2[1 - F_1(\hat{s})] + \lambda^2 f_1(\hat{s})\right] \times \left[1 - F_1(\hat{s}) + \hat{s} f_1(\hat{s})\right]$$
$$+ \frac{1}{4}\lambda^2 \int_{\hat{s}}^1 [f_1(s) - f_1(\hat{s})] f_1(s) ds.$$

If $\hat{s} \notin \hat{S}(\alpha)$, then $w = \frac{1}{2}$ cannot be sustained. Define

$$\mathcal{W}(\hat{s}; \alpha) := \{w | k(\hat{s}, w; \alpha) \geq 0 \,\&\, w \in [0,1]\},$$

which is the set of wages that can induce $\hat{s}$ without violating the nonnegativity constraint for k. It is clear that $\pi(\hat{s}, w^*; \alpha) \leq \pi(\hat{s}, \frac{1}{2}; \alpha) \equiv \bar{\pi}(\hat{s}; \alpha)$ for all $\hat{s} \in [0,1]$. □

Anti-Entrenchment For $\hat{s} \in [0, \frac{1}{2}]$, the expected profit can be bounded above by

$$\pi(\hat{s}, w; \alpha) \leq \bar{\pi}(\hat{s}; \alpha) \equiv \frac{1}{16}\left[(1-\lambda)^2[1 - F_1(\hat{s}; \alpha)] + \lambda^2 f_1(\hat{s}; \alpha)\right] \times \left[1 - F_1(\hat{s}; \alpha) + \hat{s} f_1(\hat{s}; \alpha)\right]$$
$$+ \frac{1}{4}\lambda^2 \int_{\hat{s}}^1 [f_1(s; \alpha) - f_1(\hat{s}; \alpha)] f_1(s; \alpha) ds$$
$$\leq \frac{1}{16}\left[(1-\lambda)^2 + \lambda^2\right]\left[1 - F_1(\hat{s}; \alpha) + \hat{s} f_1(\hat{s}; \alpha)\right] + \frac{1}{4}\lambda^2 \int_{\hat{s}}^1 \left[f_1(s; \alpha)\right]^2 ds$$
$$< \frac{3}{32}\left[(1-\lambda)^2 + \lambda^2\right] + \frac{1}{4}\lambda^2 \int_0^1 \left[f_1(s; \alpha)\right]^2 ds,$$

where the first inequality and second inequality follow from $f_1(s; \alpha) \in [0,1]$ for all $s \in [0, \frac{1}{2}]$. Next, it can be verified that

$$\lim_{\alpha\to\infty} \frac{1}{4}\lambda^2 \int_0^1 \left[f_1(s; \alpha)\right]^2 ds = \frac{1}{4}\lambda^2 \left[\lim_{\alpha\to\infty} \int_0^{\frac{1}{2}} \left[f_1(s; \alpha)\right]^2 ds + \lim_{\alpha\to\infty} \int_{\frac{1}{2}}^1 \left[f_1(s; \alpha)\right]^2 ds\right]$$
$$= \frac{1}{4}\lambda^2 \int_{\frac{1}{2}}^1 4 ds = \frac{1}{2}\lambda^2.$$

Therefore, it remains to find $\hat{s} > \frac{1}{2}$ and $w \in [0,1]$ that satisfy the nonnegativity condition for k, and generate a profit no less than $\frac{3}{32}[(1-\lambda)^2 + \lambda^2] + \frac{1}{2}\lambda^2$. It follows from equation (15) that the limited liability constraint for k is equivalent to

$$\frac{f_1(\hat{s}; \alpha)}{f_1(\hat{s}; \alpha) + f_0(\hat{s}; \alpha)}\left[(1-\lambda)e_1 + \lambda e_2(\hat{s})\right](1-w) \leq \underline{\pi}(e_1).$$

Equations (7), (8), and (9), together with the facts that $\frac{f_1(\hat{s};\alpha)}{f_1(\hat{s};\alpha)+f_0(\hat{s};\alpha)} < 1$ and $\underline{\pi}(e_1) \geq \frac{1}{2}(1-\lambda)e_1$, imply that it suffices to find a tuple $(w, \hat{s}) \in [0,1] \times (\frac{1}{2}, 1]$ to satisfy the following condition:

$$\frac{1 - F_1(\hat{s}; \alpha)}{4}(1-\lambda)^2 w \geq \left\{\frac{1 - F_1(\hat{s}; \alpha)}{2}(1-\lambda)^2 + \frac{f_1(\hat{s}; \alpha)}{2}\lambda^2\right\} w(1-w).$$

Assuming that it is optimal for the board to induce zero effort from the replacement manager with the constructed tuple $(w, \hat{s})$, from equation (18), we have the following additional condition:

$$\frac{1 - F_1(\hat{s}; \alpha)}{2}(1 - \lambda)w \geq \frac{1}{2}\frac{\lambda^2}{1 - \lambda}.$$

Let $\psi := (\frac{\lambda}{1-\lambda})^2$. The above two inequalities can be rewritten as

$$\frac{1 - F_1(\hat{s}; \alpha)}{2} \geq \left\{1 - F_1(\hat{s}; \alpha) + f_1(\hat{s}; \alpha)\psi\right\}(1 - w), \tag{A.6}$$

and

$$\left[1 - F_1(\hat{s}; \alpha)\right] w \geq \psi. \tag{A.7}$$

Lemma A1 states that fixing $\hat{s} > \frac{1}{2}$, $\frac{1-F_1(\hat{s};\alpha)}{2}$ can be arbitrarily close to $1 - \hat{s}$ when $\alpha \to \infty$. Similarly, it follows from Assumption 4 that fixing $\hat{s} > \frac{1}{2}$, $f_1(\hat{s})$ can be arbitrarily close to 2 when $\alpha \to \infty$. Thus, letting $\alpha \to \infty$, conditions (A.6) and (A.7) can be combined and simplified as

$$w \geq \max\left\{\frac{(1 - \hat{s}) + 2\psi}{2(1 - \hat{s}) + 2\psi}, \frac{\psi}{2(1 - \hat{s})}\right\}. \tag{A.8}$$

Next, note that the board's expected profit from the contract (w, k) that induces cutoff signal $\hat{s}$ as $\alpha \to \infty$ can be bounded below by

$$\begin{aligned}
\lim_{\alpha\to\infty} \pi(\hat{s}, w) &= w(1 - w) \times \lim_{\alpha\to\infty}\left\{(1 - \lambda)^2\left(\frac{1 - F_1(\hat{s})}{2}\right)^2 + \lambda^2\int_{\hat{s}}^{1}\left(\frac{f_1(s)}{f_1(s) + f_0(s)}\right)^2 d\frac{F_1(s) + F_0(s)}{2}\right.\\
&\quad\left. + \frac{F_1(\hat{s}) + F_0(\hat{s})}{2} \times \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \times \left[(1 - \lambda)^2\frac{1 - F_1(\hat{s})}{2} + \lambda^2\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}\right]\right\}\\
&= w(1 - w) \times \lim_{\alpha\to\infty}\left\{\frac{1}{4}\left[(1 - \lambda)^2[1 - F_1(\hat{s})] + \lambda^2 f_1(\hat{s})\right] \times \left[1 - F_1(\hat{s}) + \hat{s}f_1(\hat{s})\right]\right.\\
&\quad\left. + \lambda^2\int_{\hat{s}}^{1}[f_1(s) - f_1(\hat{s})]f_1(s)ds\right\}\\
&\geq w(1 - w) \times \lim_{\alpha\to\infty}\left\{\frac{1}{4}\left[(1 - \lambda)^2[1 - F_1(\hat{s})] + \lambda^2 f_1(\hat{s})\right] \times \left[1 - F_1(\hat{s}) + \hat{s}f_1(\hat{s})\right]\right\}\\
&= w(1 - w)\left[(1 - \lambda)^2(1 - \hat{s}) + \lambda^2\right].
\end{aligned}$$

Therefore, it remains to find $(w, \hat{s})$ such that

$$w(1 - w)\left[(1 - \lambda)^2(1 - \hat{s}) + \lambda^2\right] \geq \frac{3}{32}\left[(1 - \lambda)^2 + \lambda^2\right] + \frac{1}{2}\lambda^2,$$

which is equivalent to

$$w(1 - w)\left[(1 - \hat{s}) + \psi\right] \geq \frac{3}{32} + \frac{19}{32}\psi$$

subject to constraint (A.8). Pick $\hat{s} = \frac{1}{2} + \kappa$ for sufficiently small $\kappa > 0$, and $w = \frac{(1-\hat{s})+2\psi}{2(1-\hat{s})+2\psi}$. The above inequality can be simplified as

$$\frac{(\frac{1}{2} - \kappa + 2\psi) \times (\frac{1}{2} - \kappa)}{2(1 - 2\kappa + 2\psi)} \geq \frac{3}{32} + \frac{19}{32}\psi. \tag{A.9}$$

For w to be well-defined, we also need to satisfy

$$\frac{(1-\hat{s})+2\psi}{2(1-\hat{s})+2\psi} = w \geq \left\{ \frac{(1-\hat{s})+2\psi}{2(1-\hat{s})+2\psi}, \frac{\psi}{2(1-\hat{s})} \right\},$$

which is equivalent to

$$\frac{\frac{1}{2}-\kappa+2\psi}{1-2\kappa+2\psi} \geq \frac{\psi}{1-2\kappa}. \tag{A.10}$$

Taking limits as $\kappa \to 0$ on all sides of (A.9) and (A.10) yields

$$\frac{1}{2}+2\psi \geq (1+2\psi)\left(\frac{3}{8}+\frac{19}{8}\psi\right) \Rightarrow \psi \leq \frac{\sqrt{233}-9}{76} \approx 0.0824,$$

and

$$\frac{\frac{1}{2}+2\psi}{1+2\psi} \geq \psi \Rightarrow \psi \leq \frac{1+\sqrt{5}}{4} \approx 0.8090.$$

Therefore, for $\psi < \frac{\sqrt{233}-9}{76}$, or equivalently, $\lambda < \sqrt{\frac{\sqrt{233}-9}{76}}/[1+\sqrt{\frac{\sqrt{233}-9}{76}}] \approx 0.2231$, we can always construct a tuple $(w, \hat{s})$ with $\hat{s} > \frac{1}{2}$ to satisfy the nonnegativity constraint for k, and generate more profit than that under any contract that induces a cutoff signal less than $\frac{1}{2}$. This completes the proof of anti-entrenchment.

Entrenchment First, notice that $k(0, \frac{1}{2}; \alpha) \geq 0$ always holds, and hence $\frac{1}{2} \in \mathcal{W}(0; \alpha)$. Therefore, it suffices to show that $\pi(\hat{s}, w; \alpha) < \bar{\pi}(0; \alpha)$ for all $\hat{s} \in [\frac{1}{2}, 1]$ and $w \in \mathcal{W}(\hat{s}; \alpha)$ if α is small enough. The expected profit $\bar{\pi}(0; \alpha)$ can be rewritten as

$$\bar{\pi}(0;\alpha) = \frac{1}{16}\left[(1-\lambda)^2+\lambda^2\right] + \frac{1}{4}\lambda^2 \int_0^1 [f_1(s)-f_1(0)]f_1(s)ds = \frac{1}{16}\left[(1-\lambda)^2+\lambda^2\right] + \mathcal{G}(0;\alpha),$$

where $\mathcal{G}(\hat{s}; \alpha) := \frac{1}{4}\lambda^2 \int_{\hat{s}}^1 [f_1(s) - f_1(\hat{s})] f_1(s) ds$. It can be verified that $\mathcal{G}(\hat{s}; \alpha)$ is strictly decreasing in $\hat{s}$ because $f_1(\hat{s})$ is strictly increasing in $\hat{s}$. To proceed, it is useful to prove the following intermediate result.

Lemma A3. *There exists $\Delta^\dagger \in (0, \frac{1}{2})$ such that $\pi(\hat{s}, w; \alpha) < \bar{\pi}(0; \alpha)$ for all $\hat{s} \in [1-\Delta^\dagger, 1]$ and $w \in \mathcal{W}(\hat{s}; \alpha)$ as $\alpha \to 0$.*

Proof. It follows from Lemma A2 and Assumption 4 that for any $\epsilon > 0$ there exists a threshold $N(\epsilon)$ such that for $\alpha < N(\epsilon)$, $1 - F_1(1-\Delta^\dagger) < \Delta^\dagger + \epsilon$ for all $\Delta^\dagger \in [0, 1]$; and $f_1(\hat{s}; \alpha) < 1 + \epsilon$ for all $\hat{s} \in [\frac{1}{2}, 1-\Delta^\dagger]$. Next, note that the expected profit upon replacement can be bounded above by

$$\begin{aligned}
\underline{\pi}(e_1;\alpha) - k &\leq \frac{1}{4}\left(\frac{1}{2}\lambda + \frac{1-\lambda}{\lambda}e_1\right)^2 = \frac{1}{16}\left(\lambda + \frac{(1-\lambda)^2}{\lambda}\left[1-F_1(\hat{s};\alpha)\right]w\right)^2 \\
&\leq \frac{1}{16}\left(\lambda + \frac{(1-\lambda)^2}{\lambda}\left[1-F_1(\hat{s};\alpha)\right]\right)^2,
\end{aligned} \tag{A.11}$$

where the first inequality follows from (12) and the fact that $\frac{1}{4}(\frac{1}{2}\lambda + \frac{1-\lambda}{\lambda}e_1)^2 \geq \frac{1}{2}(1-\lambda)e_1$ for all $e_1 \in [0, 1]$; and the second inequality follows from $w \leq 1$. Therefore, for all $\hat{s} \in [1-\Delta^\dagger, 1]$, the board's expected profit can be bounded above by

$$\begin{aligned}
\pi(\hat{s}, w; \alpha) &= \int_{\hat{s}}^1 \left\{ \frac{f_1(s;\alpha)}{f_1(s;\alpha)+f_0(s;\alpha)}\left[(1-\lambda)e_1 + \lambda e_2(s)\right](1-w) \right\} d\frac{F_1(s;\alpha)+F_0(s;\alpha)}{2} \\
&\quad + \frac{1}{2}\left[F_1(\hat{s};\alpha)+F_0(\hat{s};\alpha)\right]\left[\underline{\pi}(e_1)-k\right] \\
&\leq \frac{1}{2}\left[1-F_1(\hat{s};\alpha)\right]\left\{ \frac{1-F_1(\hat{s};\alpha)}{2}(1-\lambda)^2 + \frac{f_1(\hat{s};\alpha)}{f_1(\hat{s};\alpha)+f_0(\hat{s};\alpha)}\lambda^2 \right\} w(1-w)
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{32}\left[F_1(\hat{s};\alpha) + F_0(\hat{s};\alpha)\right]\left(\lambda + \frac{(1-\lambda)^2}{\lambda}\left[1 - F_1(\hat{s};\alpha)\right]\right)^2 + 4w(1-w)\mathcal{G}(\hat{s};\alpha) \\
&\le \frac{1}{8}\left[1 - F_1(\hat{s};\alpha)\right]\left\{\frac{1 - F_1(\hat{s};\alpha)}{2}(1-\lambda)^2 + \frac{f_1(\hat{s};\alpha)}{f_1(\hat{s};\alpha) + f_0(\hat{s};\alpha)}\lambda^2\right\} \\
&+ \frac{1}{16}\left(\lambda + \frac{(1-\lambda)^2}{\lambda}\left[1 - F_1(\hat{s};\alpha)\right]\right)^2 + \mathcal{G}(\hat{s};\alpha) \\
&\le \frac{1}{8}(\Delta^\dagger + \epsilon)\left[\frac{\Delta^\dagger + \epsilon}{2}(1-\lambda)^2 + \lambda^2\right] + \frac{1}{16}\left[\lambda + \frac{(1-\lambda)^2}{\lambda}(\Delta^\dagger + \epsilon)\right]^2 + \mathcal{G}(0;\alpha).
\end{aligned}
$$

The first inequality follows from (13), (14), and (A.11); the second inequality follows from $w(1-w) \le \frac{1}{4}$ and $F_1(\hat{s};\alpha) \le F_0(\hat{s};\alpha) \le 1$; and the third inequality follows from $1 - F_1(\hat{s};\alpha) \le 1 - \hat{s} + \epsilon \le \Delta^\dagger + \epsilon$ and $\frac{f_1(\hat{s};\alpha)}{f_1(\hat{s};\alpha)+f_0(\hat{s};\alpha)} \le 1$. Note that the last expression reduces to $\frac{1}{16}\lambda^2 + \mathcal{G}(0;\alpha)$ as $\Delta^\dagger \downarrow 0$ and $\epsilon \downarrow 0$. Moreover, we have that

$$
\frac{1}{16}\lambda^2 + \mathcal{G}(0;\alpha) < \frac{1}{16}\left[(1-\lambda)^2 + \lambda^2\right] + \mathcal{G}(0;\alpha) \equiv \bar{\pi}(0;\alpha).
$$

Therefore, we can always find sufficiently small $\Delta^\dagger$ and ϵ such that

$$
\frac{1}{8}(\Delta^\dagger + \epsilon)\left[\frac{\Delta^\dagger + \epsilon}{2}(1-\lambda)^2 + \lambda^2\right] + \frac{1}{16}\left[\lambda + \frac{(1-\lambda)^2}{\lambda}(\Delta^\dagger + \epsilon)\right]^2 + \mathcal{G}(0;\alpha) < \frac{1}{16}\left[(1-\lambda)^2 + \lambda^2\right] + \mathcal{G}(0;\alpha).
$$

This completes the proof. □

Now we can prove the entrenchment result. Lemma A3 states that $s \in [1-\Delta^\dagger, 1]$ cannot be equilibrium under the optimal contract as $\alpha \to 0$, and hence it remains to show that this is the also case for all $s \in [\frac{1}{2}, 1-\Delta^\dagger]$ as $\alpha \to 0$. Recall that $\bar{\pi}(\hat{s};\alpha)$ is the maximum expected profit without the limited liability constraint for k, implying that $\pi(\hat{s}, w;\alpha) \le \bar{\pi}(\hat{s};\alpha)$ for all $\hat{s} \in [0,1]$ and $w \in [0,1]$. Moreover, from the proof of Lemma A3, we have that fixing any $\epsilon > 0$, there exists a threshold $N(\epsilon)$ such that for $\alpha < N(\epsilon)$: (i) $1 - F_1(1-\Delta^\dagger) < \Delta^\dagger + \epsilon$ for all $\Delta^\dagger \in [0,1]$; (ii) $f_1(\hat{s};\alpha) < 1 + \epsilon$ for all $\hat{s} \in [\frac{1}{2}, 1-\Delta^\dagger]$. Therefore, the expected profit for $\alpha < N(\epsilon)$ can be bounded above by

$$
\begin{aligned}
\bar{\pi}(\hat{s};\alpha) &= \frac{1}{16}\left[(1-\lambda)^2[1 - F_1(\hat{s};\alpha)] + \lambda^2 f_1(\hat{s})\right] \times \left[1 - F_1(\hat{s};\alpha) + \hat{s} f_1(\hat{s};\alpha)\right] \\
&\quad + \frac{1}{4}\lambda^2 \int_{\hat{s}}^1 [f_1(s;\alpha) - f_1(\hat{s};\alpha)] f_1(s;\alpha) ds \\
&\le \frac{1}{16}\left[(1-\lambda)^2(1-\hat{s}+\epsilon) + \lambda^2(1+\epsilon)\right] \times [(1-\hat{s}+\epsilon) + \hat{s}(1+\epsilon)] + \mathcal{G}(\hat{s};\alpha) \\
&< \frac{1}{16}\left[(1-\lambda)^2\left(\frac{1}{2}+\epsilon\right) + \lambda^2(1+\epsilon)\right] \times [1+2\epsilon] + \mathcal{G}(0;\alpha), \text{ for all } \hat{s} \in \left[\frac{1}{2}, 1-\Delta^\dagger\right],
\end{aligned}
$$

where the last inequality follows from $\hat{s} \in [\frac{1}{2}, 1]$ and $\mathcal{G}(\hat{s};\alpha) < \mathcal{G}(0;\alpha)$. Therefore, it suffices to find a sufficiently small ϵ such that

$$
\frac{1}{16}\left[(1-\lambda)^2(\frac{1}{2}+\epsilon) + \lambda^2(1+\epsilon)\right] \times [1+2\epsilon] + \mathcal{G}(0;\alpha) < \frac{1}{16}\left[(1-\lambda)^2 + \lambda^2\right] + \mathcal{G}(0;\alpha),
$$

which is obvious. This completes the proof.

Proof of Proposition 3. Given (w_1, w_2, k) and belief about period-1 effort e_1, board's optimal wage to the replacement manager can be derived as

$$
w_r(w_2, e_1) = \max\left\{\frac{1-w_2}{2} - \frac{1-\lambda}{\lambda^2}e_1, 0\right\}. \tag{A.12}
$$

Therefore, the board's expected profit after replacement is

$$\underline{\pi}(w_2, e_1) = \begin{cases} \frac{1}{4}\left(\frac{1-w_2}{2}\lambda + \frac{1-\lambda}{\lambda}e_1\right)^2 & \text{for } e_1 \leq \frac{1-w_2}{2}\frac{\lambda^2}{1-\lambda}, \\ \frac{1}{2}(1-\lambda)e_1(1-w_2) & \text{for } e_1 > \frac{1-w_2}{2}\frac{\lambda^2}{1-\lambda}. \end{cases}$$

Fixing (w_1, w_2, k) and belief about $\hat{s}$, the incumbent manager chooses e_1 and $e_2(s)$ according to

$$e_1(\hat{s}; w_1, w_2) = \frac{1}{2}\left[1 - F_1(\hat{s})\right] w_1 + \frac{1}{4}\left[F_1(\hat{s}) + F_0(\hat{s})\right] w_2, \tag{A.13}$$

and

$$e_2(s; w_1) = \frac{f_1(s)}{f_1(s) + f_0(s)}\lambda w_1. \tag{A.14}$$

If $\hat{s} \in (0, 1)$, the board's equilibrium replacement policy $\hat{s}(e_1, e_2(s); w_1, w_2, k)$ solves

$$\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}\left[(1-\lambda)e_1 + \lambda e_2(\hat{s})\right](1 - w_1) = \underline{\pi}(w_2, e_1) - k. \tag{A.15}$$

Denote the board's optimal contract by (w_1^*, w_2^*, k^*) and the corresponding equilibrium cutoff by $\hat{s}^*$. It is useful to first prove the following lemma. □

Lemma A4. *Suppose that $\lambda \in [0, 1)$, then $\hat{s}^* \neq 0$.*

Proof. Suppose to the contrary that $\hat{s}^* = 0$. Consider an alternative contract (w_1', w_2', k') that induces a positive cutoff $\hat{s}' > 0$, where

$$w_1' = w_2' = w_1^*, \tag{A.16}$$

$$k' = \underline{\pi}(w_2', e_1(\hat{s}'; w_1', w_2')) - \frac{f_1(\hat{s}')}{f_1(\hat{s}') + f_0(\hat{s}')}\left[(1-\lambda)e_1(\hat{s}'; w_1', w_2') + \lambda e_2(\hat{s}'; w_2')\right](1 - w_1'). \tag{A.17}$$

Notice that $\underline{\pi}(w_2', e_1(\hat{s}'; w_1', w_2')) > 0$ and $\frac{f_1(\hat{s}')}{f_1(\hat{s}')+f_0(\hat{s}')}$ can be arbitrarily small as $\hat{s}' \to 0$. Therefore, there exists $\hat{s}' \in (0, 1)$ such that $k' \geq 0$, and hence the constructed contract (w_1', w_2', k') is well-defined. Next, notice that

$$e_1(\hat{s}^*; w_1^*, w_2^*) = \frac{1}{2}w_1^* < \frac{1}{2}[1 - F_1(\hat{s}')]w_1^* + \frac{1}{4}\left[F_1(\hat{s}') + F_0(\hat{s}')\right] w_1^* = e_1(\hat{s}'; w_1', w_2'),$$

where the first equality follows from the postulated $\hat{s}^* = 0$ and the strict inequality follows from the fact that $F_0(\hat{s}') > F_1(\hat{s}')$ for $\hat{s}' \in (0, 1)$. Therefore, we must have that

$$\begin{aligned}
&\int_0^1 \left\{\frac{f_1(s)}{f_1(s) + f_0(s)}\left[(1-\lambda)e_1(\hat{s}^*; w_1^*, w_2^*) + \lambda e_2(s; w_1^*)\right](1 - w_1^*)\right\} d\frac{F_1(s) + F_0(s)}{2} \\
&= \int_0^{\hat{s}'} \left\{\frac{f_1(s)}{f_1(s) + f_0(s)}\left[(1-\lambda)e_1(\hat{s}^*; w_1^*, w_2^*) + \lambda e_2(s; w_1^*)\right](1 - w_1^*)\right\} d\frac{F_1(s) + F_0(s)}{2} \\
&\quad + \int_{\hat{s}'}^1 \left\{\frac{f_1(s)}{f_1(s) + f_0(s)}\left[(1-\lambda)e_1(\hat{s}^*; w_1^*, w_2^*) + \lambda e_2(s; w_1^*)\right](1 - w_1^*)\right\} d\frac{F_1(s) + F_0(s)}{2} \\
&< \int_0^{\hat{s}'} \left\{\frac{f_1(s)}{f_1(s) + f_0(s)}\left[(1-\lambda)e_1(\hat{s}'; w_1', w_2') + \lambda e_2(s; w_1')\right](1 - w_1')\right\} d\frac{F_1(s) + F_0(s)}{2} \\
&\quad + \int_{\hat{s}'}^1 \left\{\frac{f_1(s)}{f_1(s) + f_0(s)}\left[(1-\lambda)e_1(\hat{s}'; w_1', w_2') + \lambda e_2(s; w_1')\right](1 - w_1')\right\} d\frac{F_1(s) + F_0(s)}{2}
\end{aligned}$$

$$< \int_{\hat{s}'}^{1} \left\{ \frac{f_1(s)}{f_1(s) + f_0(s)} \left[(1-\lambda) e_1(\hat{s}'; w_1', w_2') + \lambda e_2(s; w_1') \right] (1 - w_1') \right\} d\frac{F_1(s) + F_0(s)}{2}$$

$$+ \frac{1}{2} \left[F_1(\hat{s}') + F_0(\hat{s}') \right] \left[\underline{\pi}(w_2', e_1(\hat{s}'; w_1', w_2')) - k' \right].$$

The first inequality follows from (A.16) and $e_1(\hat{s}^*; w_1^*, w_2^*) < e_1(\hat{s}'; w_1', w_2')$, and the second inequality follows from (A.17) and Assumption 1. The above inequality implies that the constructed contract (w_1', w_2', k') generates strictly higher profit than (w_1^*, w_2^*, k^*) to the board, a contradiction. This completes the proof. □

Now we can prove Proposition 3. First, we show that $k^* = 0$. Suppose to the contrary that $k^* > 0$. From Lemma A4, it suffices to consider the following two cases depending on the value of the equilibrium cutoff $\hat{s}^*$.

***Case I*: $\hat{s}^* \in (0, 1)$** Define

$$\Xi(w_2) := \underline{\pi}(w_2, e_1(\hat{s}^*; w_1^*, w_2)) - \frac{f_1(\hat{s}^*)}{f_1(\hat{s}^*) + f_0(\hat{s}^*)} \left[(1-\lambda) e_1(\hat{s}^*; w_1^*, w_2) + \lambda e_2(\hat{s}^*; w_1^*) \right] (1 - w_1^*).$$

It follows immediately that $k^* = \Xi(w_2^*)$ and $\Xi(1) < 0$. Therefore, there exists $w_2' > w_2^*$ such that $\Xi(w_2') = k^*/2$. Consider an alternative contract $(w_1^*, w_2', k^*/2)$. It is clear that replacement policy $\hat{s}^*$ can be induced under the alternative contract from equation (A.15). Moreover, $e_1(\hat{s}^*; w_1^*, w_2^*) > e_1(\hat{s}^*; w_1^*, w_2')$ from equation (A.13), that is, period-1 effort under the alternative contract is higher than that under contract (w_1^*, w_2^*, k^*). Therefore, we have that

$$\frac{f_1(\hat{s}^*)}{f_1(\hat{s}^*) + f_0(\hat{s}^*)} \left[(1-\lambda) e_1(\hat{s}^*; w_1^*, w_2') + \lambda e_2(\hat{s}^*; w_1^*) \right] (1 - w_1^*)$$

$$> \frac{f_1(\hat{s}^*)}{f_1(\hat{s}^*) + f_0(\hat{s}^*)} \left[(1-\lambda) e_1(\hat{s}^*; w_1^*, w_2^*) + \lambda e_2(\hat{s}^*; w_1^*) \right] (1 - w_1^*),$$

which implies that the board's expected profit after replacement under the alternative contract is again higher than that under contract (w_1^*, w_2^*, k^*). Therefore, the alternative contract $(w_1^*, w_2', k^*/2)$ generates a higher profit to the board than contract (w_1^*, w_2^*, k^*), a contradiction.

***Case II*: $\hat{s}^* = 1$** It follows that the left-hand side of (A.15) is less than the right-hand side at $\hat{s} = 1$. In this case, the board can simply decrease the severance pay to 0 without violating any constraints and strictly increase the expected profit.

Next, we show that $w_1^* = w_2^*$ and $\hat{s}^* = \frac{1}{2}$ if $\lambda = 0$. The board's profit maximization problem can be written as

$$\max_{\{w_1, w_2, e_1, \hat{s}\}} \pi(w_1, w_2, e_1, \hat{s}) := \frac{1}{2} \left[1 - F_1(\hat{s}) \right] e_1 (1 - w_1) + \frac{1}{4} \left[F_1(\hat{s}) + F_0(\hat{s}) \right] e_1 (1 - w_2)$$

s.t.

$$e_1 - \left\{ \frac{1}{2} \left[1 - F_1(\hat{s}) \right] w_1 + \frac{1}{4} \left[F_1(\hat{s}) + F_0(\hat{s}) \right] w_2 \right\} = 0, \tag{A.18}$$

and

$$\frac{1}{2}(1 - w_2) - \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}(1 - w_1) = 0. \tag{A.19}$$

Let $\mathcal{L}$ be the Lagrangian and denote λ_1 and λ_2 as Lagrangian multipliers on constraints (A.18) and (A.19), respectively. The first-order conditions for w_1 and w_2 yield the following:

$$\frac{\partial \mathcal{L}(w_1, w_2, e_1, \hat{s}, \lambda_1, \lambda_2)}{\partial w_1} = 0 \Rightarrow -\frac{1}{2} \left(e_1 + \lambda_1 \right) \left[1 - F_1(\hat{s}) \right] + \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \lambda_2 = 0.$$

$$\frac{\partial \mathcal{L}(w_1, w_2, e_1, \hat{s}, \lambda_1, \lambda_2)}{\partial w_2} = 0 \Rightarrow -\frac{1}{2}(e_1 + \lambda_1) \left[F_1(\hat{s}) + F_0(\hat{s}) \right] - \lambda_2 = 0.$$

Lemma A4 states that inducing $\hat{s} = 0$ is never optimal to the board. Moreover, $\hat{s} = 1$ is not optimal because this cutoff induces zero period-1 effort. Thus, we have that $\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} > 0$, $F_1(\hat{s}) + F_0(\hat{s}) > 0$ and $1 - F_1(\hat{s}) > 0$. This indicates that $\lambda_2 = 0$ and

$\lambda_1 = -e_1$. The first-order condition of the Lagrangian with respect to $\hat{s}$ yields

$$\frac{\partial \mathcal{L}(w_1, w_2, e_1, \hat{s}, \lambda_1, \lambda_2)}{\partial \hat{s}} = 0$$

$$\Rightarrow -e_1(1-w_1)f_1(\hat{s}) + \frac{1}{2}\left[f_1(\hat{s}) + f_0(\hat{s})\right] e_1(1-w_2) + \lambda_1 \left(f_1(\hat{s})w_1 - \frac{1}{2}\left[f_1(\hat{s}) + f_0(\hat{s})\right] w_2 \right) = 0$$

$$\Rightarrow \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} = \frac{1}{2} \Rightarrow \hat{s}^* = \frac{1}{2}.$$

Last, it follows from the board's indifference condition (A.19) and $\hat{s}^* = \frac{1}{2}$ that $w_1^* = w_2^*$. This completes the proof.

Proof of Proposition 4. First, fixing $\{w(s), r(s), k(s)\}$ and w_r, the replacement manager chooses e_{2r} to maximize

$$\frac{1}{2}\left[(1-\lambda)e_1 + \lambda e_{2r}\right] w_r - \frac{1}{2}e_{2r}^2 \Rightarrow e_{2r}(w_r) = \frac{1}{2}\lambda w_r.$$

Second, given contract $\{w(s), r(s), k(s)\}$, the incumbent manager chooses e_1 and $e_2(s)$ to maximize

$$\int_0^1 r(s)\left\{ \frac{f_1(s)}{f_1(s)+f_0(s)}\left[(1-\lambda)e_1 + \lambda e_2(s)\right] w(s) - \frac{1}{2}[e_2(s)]^2 \right\} d\frac{F_1(s)+F_0(s)}{2}$$
$$+ \int_0^1 [1-r(s)]\, k(s) d\frac{F_1(s)+F_0(s)}{2} - \frac{1}{2}e_1^2.$$

The first-order conditions with respect to e_1 and $e_2(s)$ yield

$$e_1 = (1-\lambda)\int_0^1 r(s)\frac{f_1(s)}{f_1(s)+f_0(s)} w(s) d\frac{F_1(s)+F_0(s)}{2},$$

and

$$e_2(s) = \lambda \frac{f_1(s)}{f_1(s)+f_0(s)} w(s).$$

It is clear from the above two equations that $k(s)$ cannot provide incentive on either e_1 or $e_2(s)$. Moreover, because the signal is contractible, the board can directly condition replacement on it. Therefore, we must have $k^*(s) = 0$ for all $s \in [0,1]$ in the optimal contract.

Finally, the board chooses $\{w(s), r(s)\}$ and w_r to maximize

$$\int_0^1 r(s)\left\{ \frac{f_1(s)}{f_1(s)+f_0(s)}\left[(1-\lambda)e_1 + \lambda e_2(s)\right][1-w(s)] \right\} d\frac{F_1(s)+F_0(s)}{2}$$
$$+ \int_0^1 [1-r(s)]\left\{ \frac{1}{2}\left[(1-\lambda)e_1 + \lambda e_{2r}\right]\left[1-w_r\right] \right\} d\frac{F_1(s)+F_0(s)}{2}.$$

Fixing e_1, pointwise maximization on w_r and $w(s)$ yield

$$w_r(e_1) = \max\left\{ \frac{1}{2} - \frac{1-\lambda}{\lambda^2}e_1, 0 \right\},$$

and

$$w(s, e_1) = \max\left\{ \frac{1}{2} - \frac{1}{2}\frac{f_1(s)+f_0(s)}{f_1(s)}\frac{1-\lambda}{\lambda^2}e_1, 0 \right\}.$$

Therefore, the board's expected profit can be rewritten as

$$\int_0^1 r(s)\pi_1(s, e_1) d\frac{F_1(s)+F_0(s)}{2} + \int_0^1 [1-r(s)]\, \underline{\pi}(e_1) d\frac{F_1(s)+F_0(s)}{2}$$

$$= \int_0^1 \left\{ r(s)\pi_1(s,e_1) + [1-r(s)]\,\underline{\pi}(e_1) \right\} d\,\frac{F_1(s)+F_0(s)}{2},$$

where

$$\pi_1(s,e_1) := \frac{f_1(s)}{f_1(s)+f_0(s)}\left[(1-\lambda)e_1 + \lambda^2 \frac{f_1(s)}{f_1(s)+f_0(s)} w(s,e_1)\right]\left[1-w(s,e_1)\right],$$

and $\underline{\pi}(e_1)$ is defined in equation (7). It is straightforward to verify that $\pi_1(s,e_1)$ is strictly increasing in s and $\pi_1(\frac{1}{2},e_1) = \underline{\pi}(e_1)$. Thus, the integral is maximized by setting $r(s)=1$ for $s \in [\frac{1}{2},1]$ and $r(s)=0$ for $s \in [0,\frac{1}{2})$. This completes the proof. □